APPLICATION OF LABELING IN GRAPHS WITH CRISP AND FUZZY NATURE

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Abstract

In the study of various network problems, graph theory has turn out as a special branch for solving different issues in networking with the adoption of several graph parameters. Some graph parameters like coloring and labeling etc. have their great importance to demonstrate several issues in the region of material science, computer science, physical science, etc. If ambiguity or uncertainty occurs in graph networking problems, then the hypothesis of graph in fuzzy nature has a vital job in computer science. In the interpretation of present paper, the several applications of graph labeling that are related with graph theory in fuzzy and crisp environment has been given. Also, we have explained some application of graph labeling in networking with crisp and fuzzy nature both. Finally, the utilization of labeling graph in X-Ray Crystallography and coding of radar missile control has been described.

Keywords: Graph labeling, fuzzy labeling, networking, graceful labeling, distance labeling.

1. Introduction

Labeling in graphs is an emerging zone of graph theoretical research that has accepted a big pact of consideration. The issues associated with the labeling in graphs challenge to our intelligence for their possible solutions. Labeling of graphs was first presented in the late 1960s and furthermore the vast number of labeling techniques in graph traces their beginning in 1967 designed by Rosa [10]. The first type of labeling in graphs is named by graceful labeling which came first in the research because of Rosa [10] and then explained in the research of Golomb [2]. The idea of harmonious labeling described by Sloane and Graham [3] in their research of additive bases issues modular versions arising from error-upgrading codes.

This area has turned into an extensive area of multifaceted uses in networking, biotechnology, neural network system, and coding theory, etc. The design of graph distance labeling and their real-world application has been explained by Kumar and Pradhan [7], [8]. Gallian [1], presented a dynamic research review of labelings of graphs, and that includes various research papers with various techniques of graph labeling in the zone of graph theory. That effective survey of labelings in graphs has turn out to be a big contract in research to overcome new kinds of labeling in graph theory which is applicable in numerous real problems. A comprehensive survey on the labeling graphs with distance two has been given by Yeh [14] that usually consist of distance two labeling for distinct family of graphs with valuable results.

Since there is packed of ambiguity or uncertainty in lots of real-world systems, so fuzzy graph analysis and its importance take place in many real-life situations. To overcome the uncertainty in real world problems, Zadeh [15] developed the basic idea of Fuzzy set with several real applications in 1965. Later, in 1975, the fundamental proposal for fuzzy graphs in graph hypothesis was presented by the computer Scientist Rosenfeld [11]. Koczy [4] worked on fuzzy graphs on optimization and estimation of networks in 1992 and gives some new direction for research in this zone.

Now fuzzy graph theory has become a thrust part of investigation in graph theory. Sharma et.al [12] contributed an article that explore trends in fuzzy graphs with different fuzzy graph parameters

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occurs in many real problems. A survey of fuzzy graph theory with useful results has been written by Sunitha and Mathew [13]. This paper gives new direction to research with which we can solve many real problems. Properties of the fuzzy labeling graph in the zone of fuzzy graph have been introduced by Nagoorgani and Rajalaxmi [9]. Recently, Kumar and Pradhan [5], [6] have explained the essential proposal of distance two fuzzy labeling graphs and product fuzzy labeling graphs with some useful results.

As each and every graph parameter (crisp or fuzzy) have their significant use in real world system, so we have to work on applications for each graph parameters. The present paper also considers the real-life application on graph labelings within the environment of crisp graph theory and fuzzy graph theory both. First, some fundamental definitions of labelings in graphs with proper explanation are given and then we have explained some application of graph labeling in networking with crisp nature and fuzzy nature both. The graph labeling concept in X-Ray Crystallography and Missile (or Radar) guidance also has been explained.

2. Some Fundamental Definitions of Graph Labelings

In this section, some basic definition of labelings for fundamental crisp graph and fuzzy graph has been described. For these labeling techniques the several applications are explained in the next section.

We have used the notation of graph as $G=(N_G, A_G)$ or $G=(N(G), A(G))$ where $N(G)$ *or* N_G the notation of the node set and $A(G)$ *or* A_G the notation of the arc set.

Graph Labeling [5]: The graph labeling is defined by the allocation of labels (usually with a symbol or an integer or some unique digit) to the arcs or nodes or both for a graph. Figure 1 describes this definition.

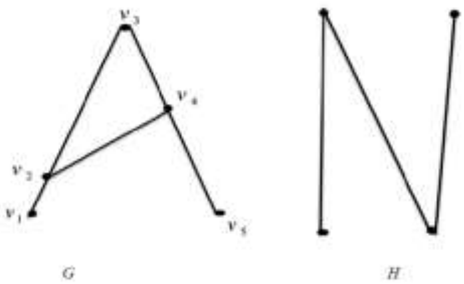

Fig.1. Labeled graph G and unlabeled graph H

Graceful Labeling [1]: Suppose $G=(N(G), A(G))$ is a graph with $|A(G)|=i$. Then a mapping $\eta : N(G) \to \{0\} \cup N_i$ named as graceful labeling if each arcs of G assign label such that $\eta(r, s) = |\eta(r) - \eta(s)| \ \forall \ r, s \in N(G)$, where $\eta(r)$ and $\eta(s)$ are node label. See graceful labeled graph in figure 2

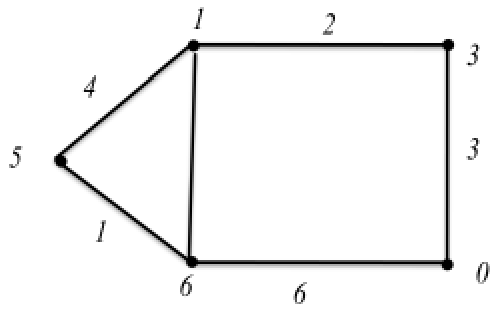

Fig.2. Graceful Labeled Graph

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Distance Labeling [8]: Let G is a graph with node set $N(G)$. Let I^+ is a set of positive integers then a mapping $\psi : N(G) \to \{0\} \cup I^+$ satisfy the condition that $|\psi(p) - \psi(q)| \geq 2$ if node p is at distance 1 from node q and $|\psi(p) - \psi(q)| \geq 1$ if node p is at distance 2 from the node q, then it explain a distance two labeling or an $L(2, 1)$-labeling for the graph G.

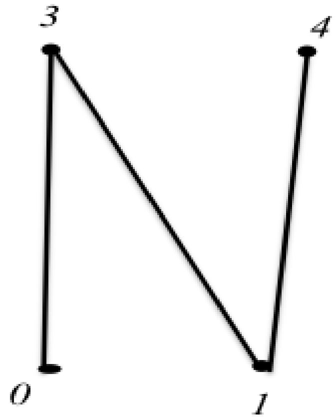

Fig.3. $L(2, 1)$-Labeling Graph

Fuzzy Labeling Graph [9]: Any fuzzy graph $\partial(G) = (\sigma_{N(G)}, \mu_{A(G)})$ with underlying crisp graph $G^* = (N(G), A(G))$ is known as a fuzzy labeling graph if membership functions $\sigma_{N(G)} : N(G) \to [0, 1]$ and $\mu_{A(G)} : N(G) \times N(G) \to [0, 1]$ gives distinct assignment labels to arcs and nodes with the condition $\mu_{A(G)}(r, s) < \min\{\sigma_{N(G)}(r), \sigma_{N(G)}(r, s)\}\ \forall\ r, s \in N(G)$. See figure 4.

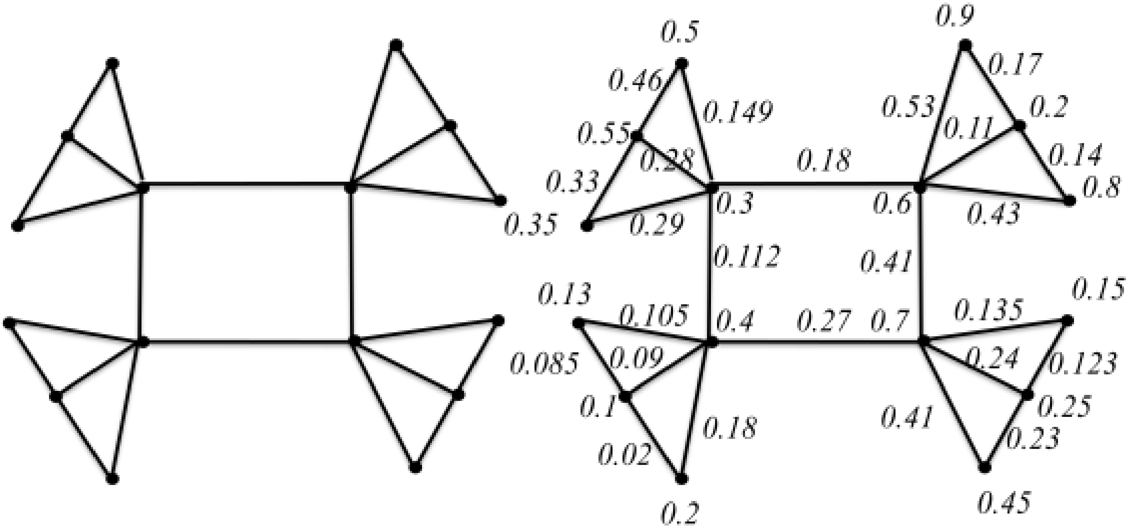

Fig.4. Left: Corona crisp underlying graph and **Right:** Corona fuzzy labeling graph

3. Application of graph labeling

In Discrete Mathematics, the study of labeling graphs has great importance. Some of the graph labeling techniques with appropriate examples have been explained in the above section. The labeling in graphs is the sole major field of analysis in graph theory because of its vast applications in many different fields of Computer Science and Technology. Some of these areas are given below:

- Coding Theory
- Communication Networks
- Circuit Design

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- Radar and Astronomy
- X-Ray Crystallography
- Image Processing
- Scheduling and Planning
- Traffic light problem

Generally, there are several graph labeling techniques that can be apply in the above-mentioned areas of research. But, some of the labeling techniques (explained in above section) are described with their significant use in the area of Networking, Crystallography, Radar and Astronomy as follows:

3.1 Application of graph labeling in networking with crisp nature

The network of communication is a collection of the network (nodes), every one of which has the calculating capability and can receive and transmit any information over communication channels (wired or wireless). Some network topologies like fully connected, mesh, star mode, ring shape, and bus shape are the collection of basic network topologies and can be graphically represented as in figure 5 given below. Communication systems are also expressed as LAN (Local Area Networks) that is inside one department or WAN (Wide Area Networks) that is among many departments.

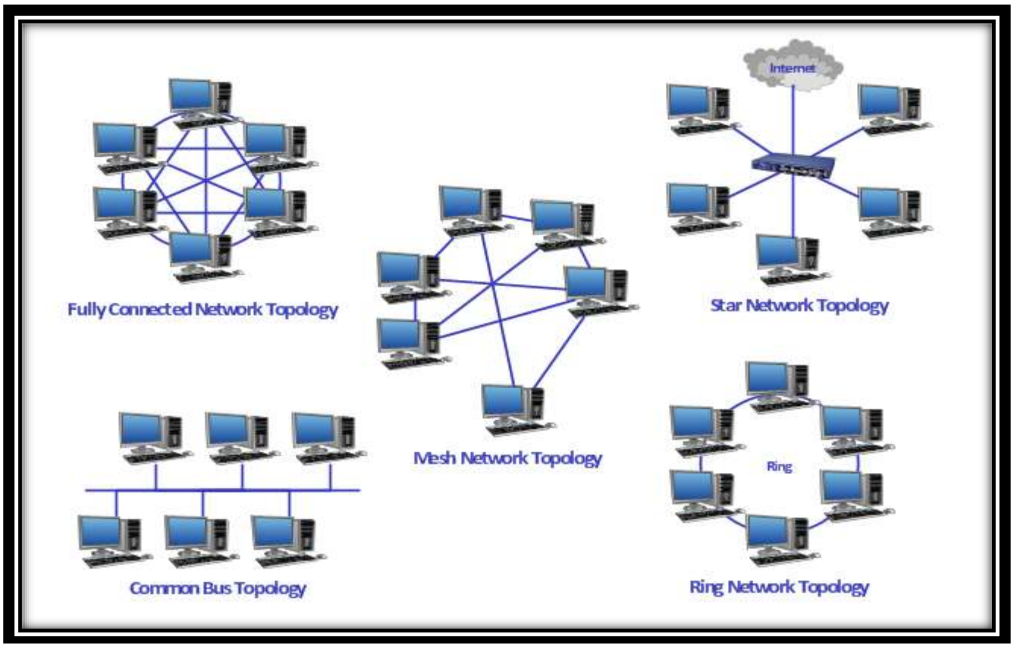

Fig.5. Graphical picture of some network topologies

For secure and correct networking of the above types of communication networks, it may be helpful to allot every client terminal a node number or label (with distance restriction) being depend on the constraints that all connecting arcs (communication joins) get different numbers. In the sound of graph theory, this secure technique of communication networking is based on graceful labeling of graphs which is described in the section 2. Therefore, the labels of any two communicating terminals naturally indicate the communication link label of the connecting path; and on the other hand, the path or arc label uniquely determines the pair of client terminals that are connected to each other.

In communication networking areas like Wi-Fi, Cellular telephone, security system, and many more, safe and secure transmissions are necessary for finding an efficient way of communication. Nowadays it is unlikable being on mobile phones and getting someone else on the same line at the same time. This trouble happens due to interferences because of uncontrolled simultaneous transmissions. Any pair of channels that are close can resonate or interfere there by the ruinous (bad) communications. This problem of interference can be fending off with the help of an appropriate channel labeling which is generally based on distance two labeling of the graph.

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The problem of channel labeling is to allocate a channel (positive number) to every communicating transmitter (like radio or television) situated at different places in a telecasting communication network for maintaining a strategic distance from unasked interference. According to the channel labeling problem in order to avoid interference, if any pair of transmitters is "close" then they must receive diverse channels and if any pair of transmitters is "very close" then they must receive channels which are at least two channels apart. To make an interpretation of this issue into the style of graph theory, we denote the representation of transmitters with nodes or vertices in a graph such that if two nodes are adjacent then they said to be "very close" and if in the graph any two nodes are at distance two apart then they said to be "close". This channel labeling issue is based on distance two labeling of graphs and the technique of distance two labeling of a graph is described in the previous section with proper explanations. At present, distance two labeling in graphs is very useful in mobile networking, dish TV networking, radio channel labeling etc. and give rise a correct solution of networking without interference.

3.2 Application of graph labeling in networking with fuzzy nature

Graph theory, no doubt, is a very important tool for representing several types of real-world problems. However, these days, graphs don't express all the real-world systems correctly because of the ambiguity or uncertainty of the system parameters. For instance, a communication network (like Gmail, Facebook) can be described by make use of a graph whose nodes delineate accounts (concerning persons) and arcs delineate the correlation amongst the accounts. In case, if the relationships between the accounts are to be estimated as weak or strong or in between according to the contact frequencies among the accounts, then in this case, fuzziness supposed to be applied for the demonstration of the resulting graph. This issue and many other difficulties cause to specify the fuzzy graphs and fuzzy graph theory. A particular communication network graph labeling in fuzzy nature has been pictured in figure 6.

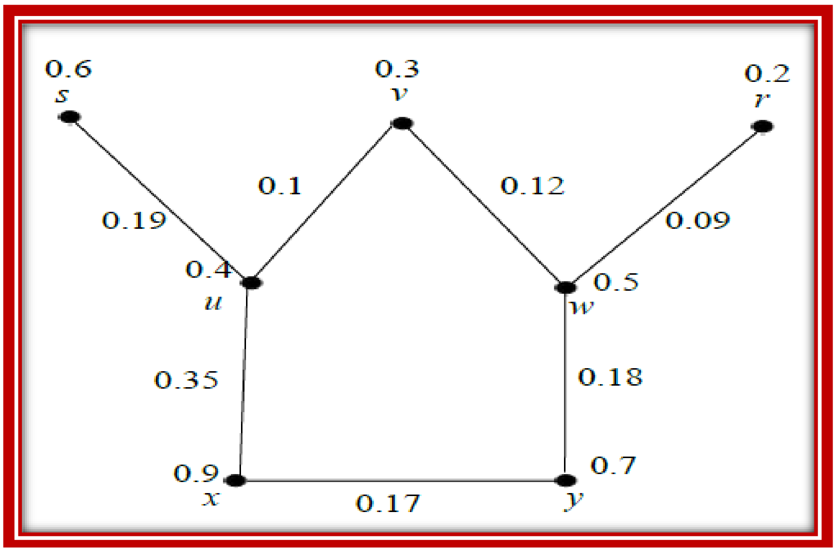

Fig.6. A communication network graph labeling in fuzzy nature

Fuzzy graph and Crisp graph both are similar in structure, but if there is uncertainty or vagueness on the nodes or arcs or both for the graph then the fuzzy graph has its own importance. Fuzzy graph labeling idea is explained in section 2 with suitable example and can be use in several networking problems that involve uncertainty. If distance restriction appears for the nodes in communication networking, then a fuzzy labeling graph with distance restriction will be the solution for such networking problems.

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3.3 Application of graph labeling in X-Ray Crystallography

The X-Ray diffraction is the main dominant method that characterized the several structural features for the crystalline solids. In the X-Ray diffraction process, a radiation beam of X-Ray or particles from the X-Ray tube collides on a crystal and produces diffraction of rays into various directions. During this procedure, more than one crystal structure gives similar diffraction data in some cases. In the language of mathematics, this X-Ray crystallography issue is convertible to deciding all labeling of the proper resulting graphs that design a pre-specified collection of arc labels which is based on graceful labeling for resulting graph.

With the thought of graph theory, in a crystal structure, the position of atoms is generally depicted by the X – ray's diffraction patterns. Estimations show the arrangement of internal atom distances in the crystal lattices.

Mathematically, corresponding to one atom position we can generate a finite set of numbers (called node labels) say $L = \{0 = m_1 < m_2 < m_3 < ..... < m_n\}$, in such a way that diffraction is equal to different arc length (i.e. difference) between these two node labels integers. This term is equivalent to graceful labeling technique. A pictorial representation of X-Ray crystallography has been given in diagram 7.

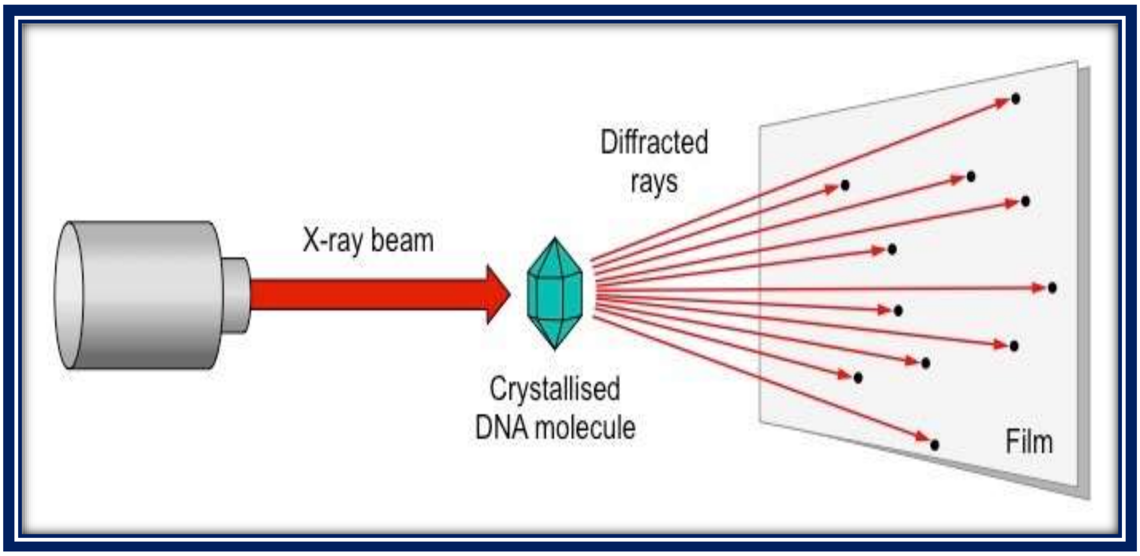

Fig.7. Graphical representation of X-Ray crystallography

3.4 Graph labeling in Radar or Missile control with coding theory

In the analysis of coding theory, the thought of graph labeling is more suitable in the proper coding for radar or missile control. The propose of a specific significant collection of proper codes (non-periodic) for missile guidance and pulse radar is proportionate to the labeling of nodes for the complete graph with the condition that each and every arc labels or assignment are not equal. The assignments of labels for nodes during this arrangement decide the different time positions at which several pulses are transmitted.

4. Conclusion

Graph labeling in fuzzy and crisp nature has become a major tool to analyze different real problems because this technique makes things excessively simple and easy. Different research papers which supposed to be the wellspring of graph labeling methods for fuzzy and crisp graphs have been studied. The principle motivation behind the exploration of this paper is to describe some graph labeling techniques and their remarkable application in the area of Communication Networking, Crystallography and Missile coding.

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In the field of computer science particularly in communication networking, the several uses of graph labeling in the crisp and fuzzy nature of graphs have been described. The utilization of graph labeling in the spectral graph of material by X-ray crystallography likewise has been explained. It is also described how the code for Missile and Radar guidance is related to graph labeling. Graph labeling in Radar or Missile control with coding theory has been explained.

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