

**A STUDY OF HEAT TRANSFER IN THE FLOW OF A SECOND-ORDER
FLUID THROUGH A CHANNEL WITH POROUS WALLS UNDER A
TRANSVERSE MAGNETIC FIELD**

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ABSTRACT

The goal of this research is to use a regular perturbation approach to investigate heat transfer in the flow of a second-order fluid through a channel with porous walls in the presence of a transverse magnetic field. For varying values of the Hartman and Reynolds numbers, the second-order effects on the temperature profile are shown. By setting the second-order parameter to zero, the findings may also be obtained for Newtonian fluids.

INTRODUCTION

In issues of gaseous diffusion and other applications, heat transfer in the flow of an electrically conducting fluid between porous barriers is of practical importance. Terrill and Shrestha looked at the problem of constant laminar flow of an incompressible viscous fluid in a two-dimensional channel with varying permeability walls, as well as the effects of a magnetic field on the fluid's electrical conductivity. Agrawal explored the problem of second-order fluid flow with heat transmission in a conduit with porous walls. Sharma and Singh investigated the numerical solution of a second-order fluid flow via a porous channel in a transverse magnetic field. The goal of this research is to use a regular perturbation approach to investigate heat transfer in the flow of a second-order fluid through a channel with porous walls in the presence of a transverse magnetic field. For varying values of the Hartman and Reynolds numbers, the second-order effects on the temperature profile are shown. By setting the second-order parameter to zero, the findings may also be obtained for Newtonian fluids.

THE PROBLEM'S FORMULATION

The heat transfer in a two-dimensional steady flow of an incompressible second-order fluid in a channel with a width of $2h$ and two porous walls of equal permeability (coinciding with the plane $y = h$) is studied. The channel's whole system is designed in such a way that the bottom and top are completely insulated and do not transmit heat. H_0 is a constant magnetic field applied normal to the channel axis. Because the magnetic Reynolds number is low, the induced magnetic field has been ignored in the flow. A consistent suction V is applied to the channel's two walls. Let's pick an x and y axis on a plane parallel and perpendicular to the channel walls, respectively. Let u and v represent the velocity components in the x and y directions, respectively.

A stream function is used to follow Terrill and Shrestha.

$$\psi(x, y) = (hU - Vx) f(\eta) \quad (1.1)$$

Where U denotes the entry velocity, $(\eta = y/h)$ denotes the dimensionless distance, and $2h$ is the distance between the channel walls. Terril and Shrestha's velocity field in non-dimensional form is as follows:

$$(U - Vx/h) f'(\eta) = U(x, y) \quad (1.2)$$

Where the dash represents a distinction with regard to. According to the formula (1.2), u is a function of x and v is a function of η alone. The constitutive equation (1.4), as well as the equations of continuity and momentum, may be expressed as follows:

$$u/x + (1/h)(v/\eta) = 0 \quad (1.3)$$

$$\begin{aligned} & \left(\frac{v}{h} \right) \frac{v}{\eta} = \frac{u}{x} + \left(\frac{v}{h} \right) \frac{v}{\eta} = \left(\frac{p}{x} \right) + \left(\frac{v}{h} \right) \frac{v}{\eta} - \left(\frac{1}{p} \right) \left(\frac{p}{x} \right) + \left(\frac{v}{h} \right) \frac{v}{\eta} \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \\ & \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \\ & \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \\ & \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \end{aligned} \quad (1.4)$$

$$-\left(\frac{1}{p} \right) \left(\frac{p}{x} \right) + \left(\frac{v}{h} \right) \frac{v}{\eta} + v^2 + \left(\frac{1}{p} \right) \left(\frac{p}{x} \right) + \left(\frac{v}{h} \right) \frac{v}{\eta} + v^2$$

$$\begin{aligned} & \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \\ & \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \\ & \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \left[\left(\frac{2u}{x} \right)^2 + v^2 \left[\left(\frac{1}{h} \right)^2 \right] \right] \end{aligned} \quad (1.5)$$

$$\begin{aligned} & k \left(\frac{2T}{x} + \frac{2T}{y} \right) + pcv \left(\frac{uT}{x} + \frac{vT}{y} \right) + pcv \left(\frac{uT}{x} + \frac{vT}{y} \right) + pcv \left(\frac{uT}{x} + \frac{vT}{y} \right) + pcv \left(\frac{uT}{x} + \frac{vT}{y} \right) + pcv \left(\frac{uT}{x} + \frac{vT}{y} \right) \\ & \left(\frac{2T}{x} + \frac{2T}{y} \right) + pcv \left(\frac{uT}{x} + \frac{vT}{y} \right) + pcv \left(\frac{uT}{x} + \frac{vT}{y} \right) + pcv \left(\frac{uT}{x} + \frac{vT}{y} \right) + pcv \left(\frac{uT}{x} + \frac{vT}{y} \right) + pcv \left(\frac{uT}{x} + \frac{vT}{y} \right) \end{aligned} \quad (1.6)$$

$\nu_1 (=1/\rho)$ is the kinematic viscosity, $\nu_2 (=2/\rho)$ is the kinematic elastic-viscosity, $\nu_3 (=3/\rho)$ is the kinematic coefficient of cross-viscosity, c_v is the specific heat at constant volume, k is the thermal conductivity, and y/h is the dimensionless distance.

The viscous dissipation function is defined as follows:

$$\Phi = \mu (u_x^2 + v_x^2 + w_x^2 + u_y^2 + v_y^2 + w_y^2 + u_z^2 + v_z^2 + w_z^2) \quad (1.7)$$

The mixed deviatoric stress tensor is denoted by τ_{ij} .

The boundary criteria are as follows:

$$\begin{aligned} u(x,0) = 0, v(x,1) = V, v(x,-1) = -V, w(x,-1) = -V \\ T(x,1) = T_1, T(x,-1) = T_2 \end{aligned} \quad (1.8)$$

We get by substituting (1.2) in equations (1.4) and (1.5) and removing p from the resulting equation.

$$\mu (u_x^2 + v_x^2 + w_x^2 + u_y^2 + v_y^2 + w_y^2 + u_z^2 + v_z^2 + w_z^2) - S_2 f'' = 0, \quad (1.9)$$

The suction is represented by $R (= Vh/\nu_1)$. The Hartmann number, $S_2 = \mu H_0^2 / (\rho \nu_1)$, is an elastic-viscous parameter guiding the effects of elastic-viscosity of the fluid, and Reynolds number, $1 (= \nu_2 V / h \nu_1)$, is an elastic-viscous parameter governing the effects of elastic-viscosity of the fluid.

The shape of the temperature distribution is suggested by equations (1.6) and (1.2) as follows:

$$T = T_1 + (\nu_1 V) [(\dots) + (U/V) - (x/h)^2] / (h c_v) \quad (1.10)$$

We derive the coefficient of $(U/V - x/h)^2$ and terms independent of $(U/V - x/h)^2$ on both sides of the resultant equation by using equation (1.10) in equation (1.6) and equating the coefficient of $(U/V - x/h)^2$ and terms independent of $(U/V - x/h)^2$ on both sides of the resulting equation.

The Prandtl number is $Pr = \mu c_p / k$, and the second-order parameter is $\beta = 2 \mu / (h^2 \rho)$.

The temperature distribution may be stated in a dimensionless manner as follows:

$$T = (T - T_1) / (T_1 - T_2) = E(1 + \eta^2), \quad T = (T - T_1) / (T_1 - T_2) = E(1 + \eta^2), \quad T = (T - T_1) / (T_1 - T_2) \quad (1.13)$$

where $E = (\nu_1 V / (T_1 - T_2) h c_p)$ is the Eckert number and $\eta = (U/V - x/h)$ is the dimensionless distance.

V.3 THE PROBLEM'S SOLUTION

Using the correlations $1 = -R$ (10) and $S_2 = R S_1^2$ eqn. (5.9),

$$\begin{aligned} (f'' - f''') F_{iv} + R (f'' - f''') \\ - R S_1^2 f'' = 0 \quad -R (f' v - f' f_{iv}) - R S_1^2 f'' = 0 \quad -R (f' v - f' f_{iv}) - R S_1^2 f'' = 0 \end{aligned} \quad (1.14)$$

We can design a regular perturbation strategy for solving eqns for modest values of the suction Reynolds number R . (1.11), (1.12), and (1.14) are obtained by multiplying f , $-\frac{1}{R}$, and $-\frac{1}{R^2}$ in R powers. Substituting $f = f_0 + \frac{1}{R}f_1 + \frac{1}{R^2}f_2 + \dots$ (1.15)

$$f'' + f f' = -\frac{1}{R} \quad (1.16)$$

We get the following sets of equations by combining eqns. (1.11), (1.12), and (1.14) and equating the like powers of R on the two sides of the resultant equations:

$$f_0'' = 0$$

$$f_0'(0) = f_0'(1) = f_0''(0) = f_0''(1) = 0 \quad (1.17)$$

$$f_1'' + f_0 f_1' - f_0' f_1 = -f_0 f_0'' \quad (1.18)$$

$$- \frac{1}{R} (f_1 f_0'' - f_0' f_1' - f_1' f_0'' - f_0' f_1'') = -\frac{1}{R} (f_1 f_0'' - f_0' f_1' - f_1' f_0'' - f_0' f_1'') \quad (1.19)$$

$$f_1'' + f_0 f_1' - f_0' f_1 = -\frac{1}{R} f_0 f_0''$$

$$f_1'' + f_0 f_1' - f_0' f_1 = -\frac{1}{R} f_0 f_0''$$

$$f_2'' + f_0 f_2' - f_0' f_2 - f_1 f_1' + f_0' f_1'' = -\frac{1}{R^2} f_0 f_0'' \quad (1.20)$$

$$f_2'' + f_0 f_2' - f_0' f_2 - f_1 f_1' + f_0' f_1'' = -\frac{1}{R^2} f_0 f_0''$$

$$f_2'' + f_0 f_2' - f_0' f_2 - f_1 f_1' + f_0' f_1'' = -\frac{1}{R^2} f_0 f_0''$$

$$f_2'' + f_0 f_2' - f_0' f_2 - f_1 f_1' + f_0' f_1'' = -\frac{1}{R^2} f_0 f_0''$$

$$f_2'' + f_0 f_2' - f_0' f_2 - f_1 f_1' + f_0' f_1'' = -\frac{1}{R^2} f_0 f_0'' \quad (1.20)$$

The boundary condition (5.8) might be rephrased as follows:

$$f_0(0) = f_0'(1) = f_0''(0) = 0 \quad f_1(0) = f_1'(1) = f_1''(0) = 0 \quad f_2(0) = 0$$

$$f_0(1) = 0 \quad f_0'(1) = 1$$

$$f_0(1) = 1/E = w, \quad f_1(-1) = 0, \quad f_2(-1) = 0, \quad f_3(-1) = 0, \quad f_4(-1) = 0 \quad (\text{say}),$$

$$f_1(1) = 0, \quad f_2(1) = 1, \quad \text{and } f_3(1) = 0$$

The following is the solution to equations (1.18), (1.19), and (1.20) when the boundary condition (1.21) is applied:

$$f_0(\eta) = \frac{1}{2}(3-3\eta)(\frac{1}{2})(3-3\eta)(\frac{1}{2})(3-3\eta)(\frac{1}{2})(3-3\eta)(\frac{1}{2})(3-3\eta)(\frac{1}{2})$$

$$f_1(\eta) = -\frac{1}{280}(7-3\eta^3+2\eta^2)-\frac{12}{40}(5-2\eta^3), \quad f_1(\eta) = -\frac{1}{280}(7-3\eta^3+2\eta^2)-\frac{12}{40}(5-2\eta^3),$$

$$f_2(\eta) = \frac{1}{1293600}(14 \ 11-385 \ 9+198 \ 7+876 \ 3-703 \)-\frac{1}{280} \quad f_2(\eta) = \frac{1}{1293600}(14 \ 11-385 \ 9+198 \ 7+876 \ 3-703 \)-\frac{1}{280}$$

$$\begin{aligned}
 &9\xi^3+6\xi)+S_{12}(\xi^7-3\xi^3+2\xi)\} -S_{12}(1/100800)(159+1087-947-545-2763+207) +(S_{12}/8400)(57- \\
 &215+273-11) = 0() = 0() = 0() = 0() = 0 P(1-4) = 1() = (3/2)P(1-4) \\
 &2() = 3P2383/280-85/6/10+4/4-(3/2)^2) -P(9/280) (1-4)^2 +(S_{12}/10) (1+2^6-34)-(3/5) \\
 &P^2 (1-6) P^2 (1-6) P^2 (1-6) P^2 (1-6) P^2 (1 \\
 &(w/2)(+1), 0(), 0(), 0(), 0(), 0(), 0(), 0 \\
 &(wP/40) = 1() \\
 &(10^3 - 5 - 9) - (P/2) (212 + 6 - 64-16) \\
 &\phi^2(\xi)=P^2[29\xi^{10}/840-51\xi^8/140+37\xi^6/20-9\xi^4/2-1149\xi^2/280+ \\
 &(w/40) (1391/2520-93/2+995/20-157/14 + 59) \\
 &-P[11/168-332/280+11^4/140-36/140-38/280+10/168-S_{12}(2^2/5-13^8/280+ 6/5-74/20-57/280) \\
 &+2(3-32/5-3^8/10+12^6/5-94/2)-w(71/100800-3/840+3^5/5600- 9/ 20160) +S_{12}(19/8400- 7/1680 \\
 &+5/400- 3/240)].
 \end{aligned}$$

DISCUSSIONS AND RESULTS

- (i) The values derived by Sharma and Singh for the functions f0, f1, and f2 are identical.
- (ii) The results for 2 = 0 are quite similar to those obtained by Terril and Shrestha.
- (iii) The results for S = 0 are identical to those obtained by Agarwal.

CONCLUSIONS

The fluctuation of the temperature profile for R = 0.01, 0.1, 1.0 at P = 0.4, = 0.4, E = 1, S1 = 1, 2 = -1 shows that for R = 0.1, temperature climbs up to roughly = 0.7 and then progressively falls until it reaches its value 1 at the boundary wall = 1. The temperature graph is parabolic with the vertex upward at R = 1 and reaches its greatest value in the centre of the wall gap-length, with the minimum value at the border wall = -1. Temperature rises linearly across the wall gap-length for R = 0.01, with a minimum at the boundary wall = -1 and a high at = 1. It is also obvious from this diagram that when the suction Reynolds number R increases, the temperature rises. The temperature profile for P = 0.4, = 0.4, E = 1, S1 = 1, R = 1 for 2 = 0, 0.1, 1.0 shows that the temperature graph is generally parabolic with vertex upward and reaches its greatest value in the centre of the wall gap-length with a minimum at the border wall = -1. This picture also shows that when the cross-viscous second-order parameter 2 increases, the temperature falls. The temperature graph is essentially parabolic with vertex upward and attains its greatest value at the centre of the wall gap-length with a minimum at the border wall = -1 for P = 0.4, =

0.4, $E = 1$, $R = 1$, $2 = -1$ for $S_1 = 0, 1, 2$. This graphic also shows that when the Hartman number S_1 increases, the temperature lowers.

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