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Galerkin Based finite element method for analysis of rectangular plates

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Abstract

A study of free vibration of thin isotropic rectangular plates with varied edge conditions is carried out in this paper. The natural frequencies are obtained by employing a Galerkin-based finite element approach to solve the mathematical model that regulates the plate's vibration behaviour. In this study, cubic quadrilateral serendipity sub parametric elements with twelve degrees of freedom were utilised. Despite the fact that the order of polynomial utilised is the lowest conceivable, the method's accuracy in estimating natural frequencies is proved by comparing the answer to current analytical findings. The impact of aspect ratio, number of elements, and number of sample points on the solution's correctness is also discussed.

Introduction

Plates are widely employed as structural components in a variety of areas, including aerospace, civil engineering, hydraulic engineering, containers, ships, instrumentation, and machine parts. They are exposed to dynamic loadings in service, which have a significant impact. Plate behaviour has been studied extensively using a variety of techniques. Leissa [1] produced an outstanding study on the early literature on vibration analysis of plates. Most scholars have employed traditional thin plate theory in their formulations to examine plate response, such as [1], [2], and [3], where the flexural vibration of the thin plate is described by a fourth-order partial differential equation. Because a direct solution to such an equation may be challenging, most documented solutions rely on numerical approaches such as the finite difference method [4], and the finite element method[5],[6]. Despite the fact that the Galerkin finite element technique is very powerful, simple to grasp, and useful for a wide range of engineering issues, it has received little attention in the literature. This work is dedicated to the description of the technique as well as the demonstration of its suitability for solving isotropic plate bending problems.

The governing equation that describes the flexural vibration of thin plates subjected to transverse loading, based on classical plate theory, is expressed as[2]:

$$\rho(x,y)h\frac{\partial^2 w(x,y,t)}{\partial t^2}+D\left(\frac{\partial^4 w(x,y,t)}{\partial x^4}+2\frac{\partial^4 w(x,y,t)}{\partial x^2\partial y^2}+\frac{\partial^4 w(x,y,t)}{\partial y^4}\right)=P_z(x,y,t). \quad \textbf{(1)}$$

Where, $w(x,y,t)$, is the out of plane motion in positive z -direction, P_z is the exciting load per unit area, E, h, ν and $\rho(x,y)$, are the modulus of elasticity, plate thickness, the Poisson's ratio, and density respectively. In order to obtain the natural frequencies of the plate, the exciting load P_z is set equal to zero. The flexural rigidity is expressed as

$$D=\frac{Eh^3}{12(1-\nu)}. \quad \textbf{(2)}$$

Methodology

Eq.(1), the governing equation of a vibrating thin plate, is a fourth-order partial differential equation that needs deflection and slope continuity in both the x- and y-directions, namely w, and. To put it another way, a unique solution requires at least three degrees of freedom at each node of the chosen

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element. As a result, the plate model necessitates the use of one C-continuous parameter function. where mn is the natural frequency (rad/sec), an is the plate dimension in x-direction, b is the plate dimension in y-direction, h is the plate thickness, is the material density, D is the flexural stiffness, and m and n are the number of half waves in the x and y directions, respectively. The natural frequency, normalised, may be written as It is suggested that at least nine Gauss sampling points be utilised to provide precise numerical integration across the element [8, 9]. For the polynomial described in Eq. 13, at least nine Gauss sampling sites are suggested [8, 9] in order to get precise numerical integration across the element. For the polynomial described in Eq. 13, at least three sample points in each direction are required, as well as three points in each direction and the appropriate weights, wk and wl. As a result, at least three sample points in one direction and three points in the other are required, as well as the related weights, wk and wl. Tables 1-3 provide a comparison of the normalised natural frequencies with the precise values for the simply supported plate. Table 1 shows the convergence of the six lowest vibration modes for a simply supported rectangular plate with an aspect ratio of a/b = 1. The results clearly converge to the precise solution with 64 elements, resulting in 243 degrees of freedom. Table 2 shows that increasing the number of sample points improves the rate of convergence. Table 3 shows the findings for a rectangular plate with an aspect ratio of 1.5, which show convergence towards the precise solution.

Table 1. Normalized natural frequency ϖ compared to exact results, Eq.(14), for (SSSS) Rectangular plate with aspect ration a/b = 1, and nsp = 3.

#	m	n	Exact	Elements						
				2×2	3×3	4×4	5×5	6×6	7×7	8×8
1	1	1	2	1.81	1.90	1.94	1.96	1.97	1.98	1.98
2	2	1	5	4.32	4.69	4.80	4.86	4.90	4.92	4.94
3	1	2	5	4.32	4.69	4.80	4.86	4.90	4.92	4.94
4	2	2	8	4.37	6.88	7.27	7.47	7.61	7.70	7.76
5	3	1	10	6.87	9.59	9.71	9.76	9.81	9.85	9.88
6	1	3	10	6.87	10.11	9.71	9.77	9.81	9.85	9.88

Table 2. Normalized natural frequency ϖ compared to exact results, Eq.(14), for (SSSS) Rectangular plate with aspect ration a/b = 1, and nsp = 5

#	m	n	Exact	Elements						
				2×2	3×3	4×4	5×5	6×6	7×7	8×8
1	1	1	2	1.92	1.95	1.97	1.98	1.98	1.99	1.99
2	2	1	5	4.84	5.08	5.03	5.01	5.00	4.93	5.00
3	1	2	5	4.84	5.08	5.03	5.01	5.00	4.93	5.00
4	2	2	8	6.28	7.66	7.75	7.81	7.86	7.89	7.91
5	3	1	10	6.28	10.87	10.68	10.42	10.27	10.19	10.14
6	1	3	10	7.35	10.93	10.70	10.42	10.28	10.20	10.14

The Galerkin-based finite element's usefulness for investigating the convergence of natural frequencies of isotropic thin rectangular plates with diverse edge conditions has been established. Even though a partial third order polynomial was employed to represent the lowest feasible order utilising quadrilateral serendipity pieces, this was done. One of the method's benefits is that it just requires a tiny mesh size to provide precise results. Progressive mesh refinement resulted in a significant reduction in the analytical error. In the case of refined (8x8) meshes, the results are within an acceptable and practical range. In the case of (SSSS), an increase in the number of sample points was beneficial, while in the case of (CCCC), such an increase lowered accuracy.

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Results

The error percentages were compared with those produced using (4N-16DOF) and (16N-64DOF) elements reported in reference [10] to underline the precision of the findings obtained by the Galerkin-based finite element. Tables 6 and 7 include the findings. The highest degrees of freedom employed in this study were 243, which resulted in an inaccuracy of 0.8 percent for the first mode and 1.24 percent for the sixth mode. 1024 degrees of freedom were required for the (4N-16DOF) element to achieve 0.9 percent inaccuracy in the first mode and 0.4 percent error in the sixth mode. However, 1024 degrees of freedom were required to achieve 3.6 percent error for the first mode and 1.4 percent error for the sixth mode when employing (16N-64DOF) components. It should be noted that in the current work (4N-12DOF), the GaussLegender formulation was employed to accomplish precise integration, while in the previous studies (4N-16DOF) and (16N-64DOF), the elements' matrices were produced in closed forms to prevent numerical integration mistakes.

Table 3. Normalized natural frequency ϖ compared to exact results, Eq.(14), for (SSSS) rectangular plate with a/b = 1.5 and nsp = 3

#	m	n	Exact	Elements						
				2×2	3×3	4×4	5×5	6×6	7×7	8×8
1	1	1	3.25	2.88	3.08	3.14	3.18	3.20	3.21	3.22
2	2	1	6.25	4.44	5.72	5.92	6.03	6.09	6.13	6.15
3	1	2	10	5.80	9.34	9.60	9.73	9.80	9.85	9.88
4	3	1	11.25	7.98	9.99	10.72	10.87	10.96	11.02	11.07
5	2	2	13	9.04	10.53	11.61	12.04	12.30	12.46	12.58
6	3	2	18	9.99	11.40	15.38	16.23	16.23	16.96	17.17

Table 4. Normalized natural frequency ϖ compared to classical results, Eq.(17), for (CCCC) rectangular plate with a/b = 1. and nsp = 3

#	classical	Elements						
		2×2	3×3	4×4	5×5	6×6	7×7	8×8
1	3.56	3.57	3.42	3.48	3.52	3.56	3.58	3.59
2	7.39	9.04	7.11	7.10	7.16	7.22	7.27	7.30
3	7.39	9.04	7.11	7.10	7.16	7.22	7.27	7.30
4	10.89	*	10.30	9.94	10.09	10.26	10.40	10.51
5	13.34	*	14.86	12.97	13.01	13.03	13.07	13.11
6	13.34	*	16.20	13.17	13.15	13.14	13.17	13.20

Table 5. Normalized natural frequency ϖ compared to classical results, Eq.(17), for (CCCC) rectangular plate with a/b = 1. and nsp = 5

#	classical	Elements						
		2×2	3×3	4×4	5×5	6×6	7×7	8×8
1	3.56	4.69	3.89	3.76	3.72	3.70	3.58	3.68
2	7.39	11.71	8.85	8.13	7.84	7.70	7.27	7.63
3	7.39	11.71	8.85	8.13	7.84	7.70	7.27	7.63
4	10.89	*	13.23	11.60	11.27	11.15	10.40	11.09
5	13.34	*	18.63	16.08	15.03	14.44	13.07	14.12
6	13.34	*	20.58	16.08	15.14	14.53	13.17	14.19

Conclusion

The Galerkin-based finite element's usefulness for investigating the convergence of natural frequencies of isotropic thin rectangular plates with diverse edge conditions has been established. Even though a partial third order polynomial was employed to represent the lowest feasible order utilising quadrilateral serendipity pieces, this was done. One of the method's benefits is that it just requires a tiny mesh size to provide precise results. Progressive mesh refinement resulted in a significant reduction in the analytical error. In the case of refined (8x8) meshes, the results are within an acceptable and practical range. In the case of (SSSS), an increase in the number of sample points was beneficial, while in the case of (CCCC), such an increase lowered accuracy.

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