Discrete Element Analysis of Rock Slope Stability

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Abstract

The strength properties of the rock and the geometry and strength characteristics of the discontinuities have a significant impact on the stability of rock slopes (e.g., roughness, wall strength and persistence). A rock mass's behaviour is dominated by discontinuities like faults, joints, and bedding planes since it is not a continuum. Moreover, rock slope instability is a significant risk to human activity and frequently results in financial losses, property damage (maintenance costs), injuries, or fatalities. In this study, a computer software was created to use the Discrete Element Method to do a stability analysis of a rock slope (DEM). In the current model, the rock is represented as a set of blocks held together by elasto-plastic Winkler springs. This approach, whose simulation satisfies all equilibrium and compatibility requirements, takes gradual failure into account and can locate the slip surface or unstable blocks. Many examples for the study and optimization of the stabilisation of the rock slope have been shown to show the applicability and value of the method.

Keywords: Rock slope stability, discrete element method, limit equilibrium

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1. Introduction

According to Lin et al. (2012), the geometries and strength characteristics of discontinuities (such as roughness, wall strength, and persistence) as well as the strength features of the rock have a significant impact on the stability of rock slopes. A rock mass's behaviour is dominated by discontinuities like faults, joints, and bedding planes since it is not a continuum. In general, discontinuities (presence/absence) have a significant impact on the stability of rock slopes, and their behaviour is crucial to the assessment of stability. The numerical discontinuum modelling approach has been utilised by a number of authors to examine slope stability issues [Cundall, 1987]. Using the aforementioned technique, Easki et al. (1999) built a model of a natural slope to investigate the instability brought on by excavations close to the toe [Easki et al. 1999]. Using a discrete element model, Zhang et al. (1997) investigated the dynamic behaviour of a 120 m-high rock slope of China's Three Gorges Dam ship lock. When they measured the residual displacements of the rock slope during the excavation unloading stage, they discovered good agreement between the numerical results and the field measurements [Zhang et al. 1997]. Heuze et al. (1990) illustrated the usefulness of the discrete element approach for the anal- ysis of rock mass mechanical behavior dur- ing wave propagation due to seismic events or rock blasting and concluded that although continuum codes are quite useful in simulat- ing some ground shock effects, they are not adequate for representing dynamic block mo- tion processes[Heuze et al. 1990].

It is to be emphasized that in all the above cases, use has been made of the Coulomb-slip constitutive model for joints deformations be- cause it has been a common practice to use C and φ parameters to model the behavior of discontinuities by a linear Coulomb relation.Discrete Element Method (DEM) can be used for the numerical analyses of different geo- technical problems too. It was presented first by C. S. Chang (1991), (1992), and (1994), as a new concept to investigate the bearing ca- pacity of foundations and stability of slopes and retaining walls. Kim et al. (1997) used DEM and analyzed nailed earth slopes [Kim et al., 1997]. Kveldsvik et al. (2009) explained the static and dynamic loading conditions in DEM code for high rock slopes [Kveldsvik et al., 2009]. Rathod et al. (2011) used DEM for the analysis of static and dynamic response of dam abutments with a liner Coulomb slip con- stitutive model [Rathod, Shrivastava and Rao, 2011]. Lin et al. (2012) conducted the dynam- ic analysis of rock slope based on practical seismic load and performed collapse analysis of the crack development in a rock slope [Linet al, 2012]. Kainthola et al. (2012) analyzed a 100 m high natural hill slope composed of ba- salt using the DEM code for dry and saturated conditions [Kainthola et al., 2012]. Stability analysis of Surabhi landslide in the Dehradun and Tehri located in India, was simulated nu- merically using the distinct element method by Pal et al. [Pal et al. 2012]. Shen and Abbas (2013) developed and applied the random set distinct element method in the stability analy- sis of a rock slope from China [Shen and Ab- bas, 2013].

In this method, stresses on all blocks' interfac- es are compatible with the deformations and fully satisfy the stress-displacement relation- ship without any further assumptions. This model, a slight extension of the conventional limit equilibrium analysis, permits a solution that satisfies all equilibrium and compatibility conditions.

2. Discrete Element Method (DEM)

In this method, the rock is modelled by several solid slices connected together with Winkler springs (compression, tension and shear) (Fig-ure 1) to establish a unique bounded system.Normal springs behave elasto-plastically andinduce rotational as well as normal stiffness;they do not yield in compression, but theydo in tension cut-offs. Shear springs yield atshear capacity according to Mohr-Coulombconstitutive model (Figure 2). Blocks gravityforces are applied during the analysis.

In each calculation step, while assuming equiv-alent secant stiffness for the Winkler springs,the load is increased until the spring stresses exceed the allowable values, and when this happens at a certain interface, its local fac- tor of safety is assumed as 1 and the excess stresses are redistributed among the neighbor- ing slices through the iteration process. Thiscontinues until the stresses on all interfaces are compatible with the deformations and ful- ly satisfy the stress-displacement relationship. Failure of rock joints shear springs depends on the joint's shear resistance properties.

The authors have applied the method to the stability analysis of rock slopes and developed a computer program with which they have worked out several examples to demonstrate the method's applicability.

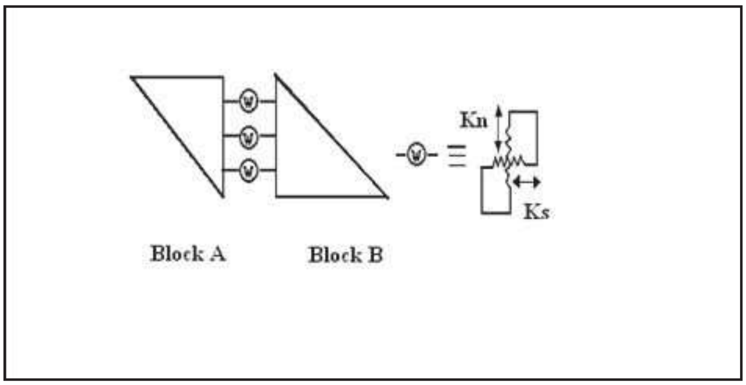

Figure 1. Winkler springs between two adjacent slices

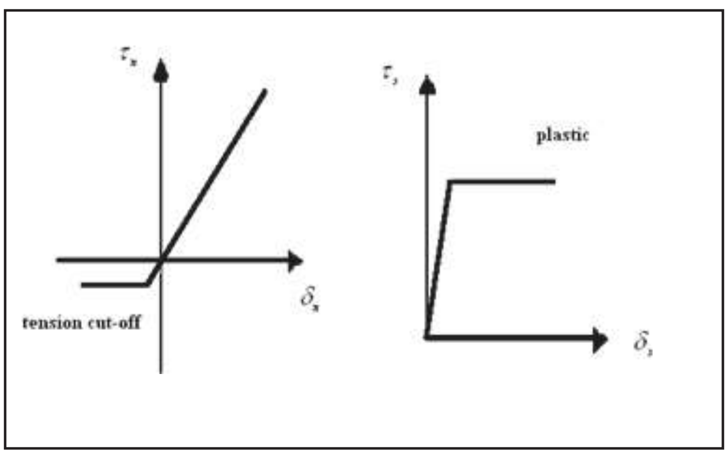

Figure 2. Behaviour of a Winkler spring

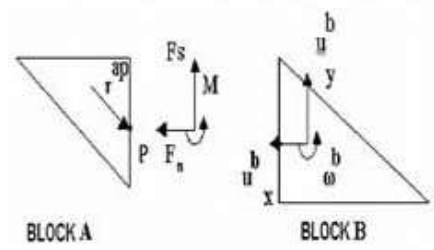

Figure 3. Two adjacent blocks

3. DEM Formulation

Consider two neighbouring blocks A and B (Figure 3). The relative displacement between the slices can be calculated using Eq. (1).

$$\begin{Bmatrix}\Delta^p_x\\ \Delta^p_y\\ \Delta^p_\omega\end{Bmatrix}=\begin{bmatrix}1 & 0 & -r^{bp}_y\\ 0 & 1 & r^{bp}_x\\ 0 & 0 & 1\end{bmatrix}\begin{Bmatrix}u^b_x\\ u^b_y\\ \omega^b\end{Bmatrix}-\begin{bmatrix}1 & 0 & -r^{ap}_y\\ 0 & 1 & r^{ap}_x\\ 0 & 0 & 1\end{bmatrix}\begin{Bmatrix}u^a_x\\ u^a_y\\ \omega^a\end{Bmatrix} \quad (1)$$

where ua , ub and ωa, ωb are the displacements and rotations of blocks A and B, respectively, P is a point located at the middle of the interface of the two blocks, and r is a vector connecting block A's centre of gravity to point P. If one of the blocks remains immobile, its displacements are set to be zero in Eq. (1).
Vector n^p (cosa , sina) is defined as an inwardunit vector normal to the face of block A atpoint P wherein a is the angle it makes with the x axis; normal to it is n^p (-sina , cosa).
Now, using Eq. (2) below, the displacement vector on the left side of Eq. (1) can be transformed from the global coordinate system (x- y) to the local coordinate system (n-s).
If k_n and k_s are respectively the normal and shear constants per unit length of the Winkler spring, the interface force between the two blocks can be calculated as follows:

$$\begin{Bmatrix}\Delta^p_n\\ \Delta^p_s\\ \Delta^p_\omega\end{Bmatrix}=\begin{bmatrix}n_x & n_y & 0\\ -n_y & n_x & 0\\ 0 & 0 & 1\end{bmatrix}\begin{Bmatrix}\Delta^p_x\\ \Delta^p_y\\ \Delta^p_\omega\end{Bmatrix}=[R]\{\Delta^p\} \quad (2)$$

$$\begin{Bmatrix}F^p_n\\ F^p_s\\ M^p\end{Bmatrix}=\begin{bmatrix}K_n & 0 & 0\\ 0 & K_s & 0\\ 0 & 0 & K_\omega\end{bmatrix}\begin{Bmatrix}\Delta^p_n\\ \Delta^p_s\\ \Delta^p_\omega\end{Bmatrix}=[K]\{\Delta^p\} \quad (3)$$

where $K_\omega = k_n L^3/12$; $K_s = k_s L$; $K_n = k_n L$ and L is the interface length.
The interface forces in the global coordinates system can be determined using Eq. (4) and the forces acting on all sides of a block can be found using Eq. (5) which is derived from the equilibrium equations.

$$\begin{Bmatrix}F^p_x\\ F^p_y\\ M^p\end{Bmatrix}=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}\begin{Bmatrix}F^p_n\\ F^p_s\\ M^p\end{Bmatrix}=[T]\{F\} \quad (4)$$

$$\begin{Bmatrix}f^a_n\\ f^a_s\\ m^a\end{Bmatrix}=\begin{bmatrix}-1 & 0 & 0\\ 0 & -1 & 0\\ r^{ap}_y & -r^{ap}_y & -1\end{bmatrix}\begin{Bmatrix}F^p_x\\ F^p_y\\ M^p\end{Bmatrix} \quad (5)$$

where f^a, f_n^a and m_s^a are respectively the body forces and the moment acting on a block, and can include gravity, inertia (earthquake) and loading forces.

Combining Equations (1) to (5) will result in Eq. (6) below which shows the relationship between the forces and displacements of a block.

$$\{f^a\} = -\sum [R^a]^T [T]^T [K][T] \left([R^b]\{u^b\} - [R^a]\{u^a\}\right) \quad (6)$$

It is obvious that for N blocks in the analysis, we have 3N equations and 3N unknown vari- ables (f_n^a, f_s^a and m^a for each block).

Failure of Winkler Springs

For each block, two internal and external load vectors are introduced; the latter is established on the basis of the loads applied to the sys- tem and the former is induced in the springs as a consequence of relative displacements between slices.

Computation convergence, a necessity when incremental loading procedure is adopted in each calculation cycle, is accomplished when internal and external load vectors become equal. As mentioned earlier, when a spring fails, its stiffness is changed by the secant method; therefore, the system does not con- verge in the first iteration. The excess stress, appearing as the difference between the inter-nal and external load vectors, is redistributed among the neighbouring slices. The iterative procedure continues until the stresses in the interfaces of all blocks are compatible with the deformations and fully satisfy the stress-displacement relationship.

Winkler Spring Stiffness

As shown in Eq. (3), the method requires kn and ks (Winkler spring's stiffness values); Chang [4] showed that they have insignificant effects on the computed results. The ratio (kn/ ks) plays an important role (similar to that of the soil's Young and shear moduli) in the analysis and is equal to 2(1+ v) for isotropic elastic materials (usually ranging from 2 to 3).

4. Computer Program

Based on the formulation described, the au- thors have developed a 2-D FORTRAN com- puter program that can analyse different-shaperock slopes with any sets of joints. The pro- gram has an interface that shows the slope's joints and the un-deformed shape (the failed joints are in red); so, the progressive failure and the deformed slope and its blocks can be observed in the program interface.

5. Model Verification

Model verification is done by comparing the program results with those of the analytical solution of the simple two-block example. Figure 3 shows a sliding surface that is joined with another flatter surface at the toe of the slope. The strength reserve in the toe (the pas- sive region resting on a relatively flat slid-ing surface) is overcome by the excess force transmitted from the upper region (the active block that cannot remain at rest by the friction along its basal surface alone).

Analysis of the force system shown in Figure 3 yields:

$$F = \frac{W_1 \sin(\delta_1 - \varphi_1)\cos(\delta_2 - \varphi_2 - \varphi_3) + W_2 \sin(\delta_2 - \varphi_2)\cos(\delta_1 - \varphi_1 - \varphi_3)}{\cos(\delta_2 - \varphi_2)\cos(\delta_1 - \varphi_1 - \varphi_3)} \quad (7)$$

where $\varphi_1, \varphi_2, \varphi_3$ are the friction angles related to slip along the upper, lower and vertical sur- faces, respectively, δ_1 and δ_2 are the inclina- tions of the upper and lower slip surfaces, respectively, W_1 and W_2 are respectively the weights of the active and passive blocks per unit of slip width, F is the thrust required in the passive block to reach limiting equilibri- um (stable, if F > 0 and unstable if F < 0).

Assuming $W_1 = 200$ t, $W_2 = 75$ t, $\delta_1 = 37^\circ$, $\delta_2 = 27^\circ$, and $\varphi_3 = 35^\circ$, we mod- elled the example by the DEM program, com- pared the results with those of the analytical solutions and found good agreement between them (Figs. 1 and 2). The red colour in each joint indicates that shear failure has occurred in the joint; this gives a good insight on the condition of each joint and helps in optimiz-ing the slope stabilization. Furthermore, the normal and shear stresses in each joint can be defined, and blocks displacements can be observed in the DEM method where stresses in all blocks interfaces are compatible with their deformations and fully satisfy the stress- displacement relationship without any further assumptions.

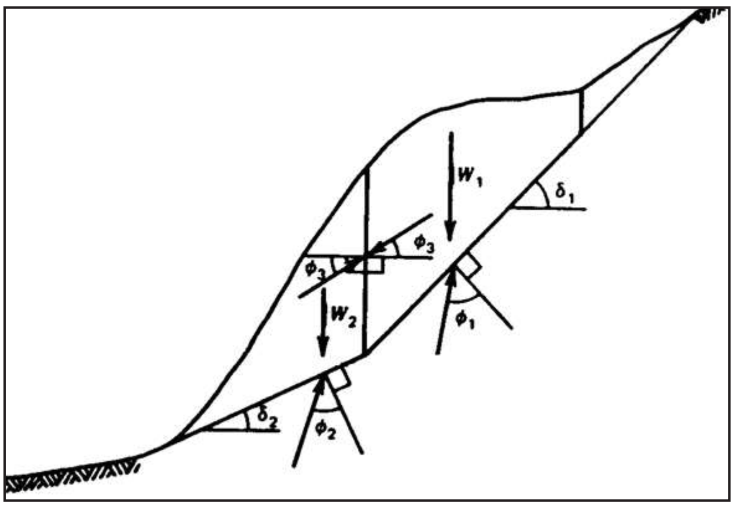

Figure 4. Model for a two-block stability analysis

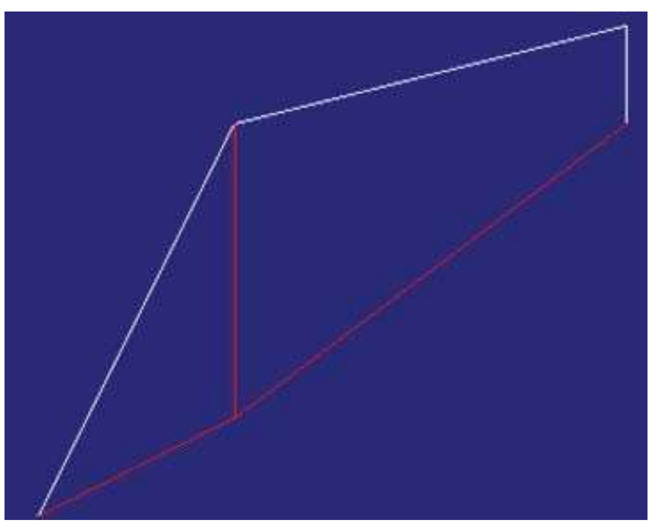

Figure 5. $\varphi_1 = 35^\circ$, $\varphi_2 = 30^\circ$ $F = 2.6$ t or $\varphi_1 = 35^\circ$, $\varphi_2 = 31^\circ$, $F = 1.2$ t

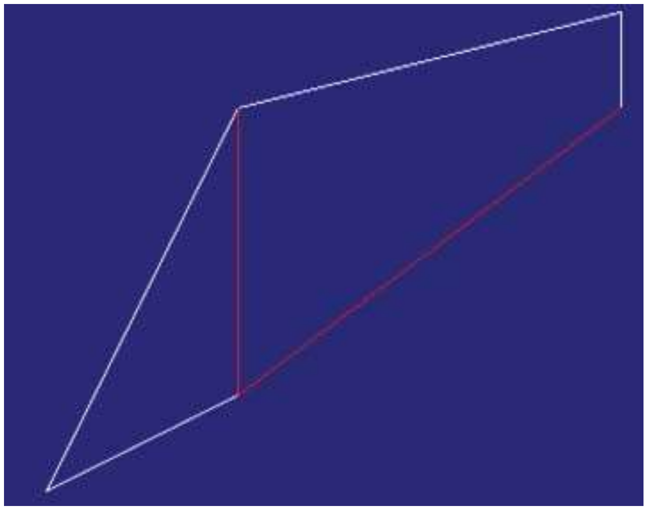

Figure 6. $\varphi_1 = 36^\circ$, $\varphi_2 = 30^\circ$, $F = -.6 \quad t$ or $\varphi_1 = 35^\circ$, $\varphi_2 = 32^\circ$, $F = -.2 \quad t$

6. Rock Slope Analysis using the DEM

Results of several examples are presented to show the DEM's applicability for the stability analysis of rock slopes and its usefulness in optimizing the slope stabilization. Examples include the stability analyses of a slope withtwo sets of perpendicular joints, a slope withinclined layers, and a toppling mode.

Stability Analysis of a Slope with Two Sets of Perpendicular Joints

This example considers a slope with two sets of perpendicular joints with the following data: H=5 m c=0 t/m^2 φ=30° y=2.6 t/m^3 Figure 7 shows the rock slope model and thejoints directions. The red colour in each jointindicates that shear failure has occurred at thejoint.

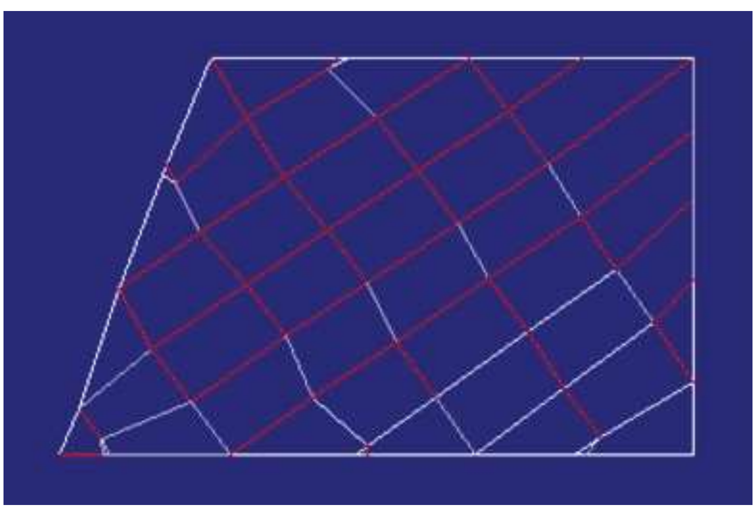

Figure 7. Stability analysis of a slope with two sets of perpendicular joints

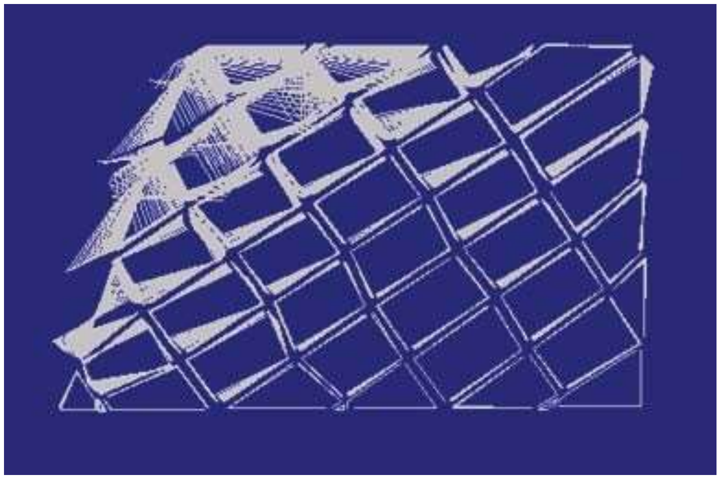

Figure 8. Rock blocks displacements in a slope with two sets of perpendicular joints

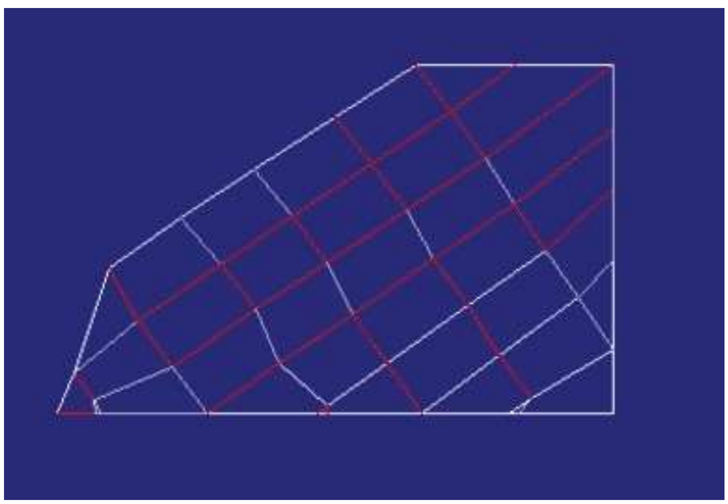

Figure 9. Stability analysis of the slope with the removal of unstable blocks (slope is stable)

Figure 8 shows blocks displacements and indicates that failure has occurred in this exam-ple. Figs. 7 and 8 help us decide how we canstabilize the slope by the removal of unstable blocks. The six upper blocks have produced a failure surface and have large displacements, so we can easily decide to remove them and re-examine the slope stability; Figure 9 shows that blocks removal has caused the slope to become stable.

Stability Analysis of a Slope with In- clined Layers

The rock slope in this analysis is the same asthat in the previous section, but with inclined layers. During the analyses, blocks weights were applied in steps. The rock slope model and joints directions are shown in Figure 10 and blocks displacements (in the steps) in Fig-ure 11.

Block-removal stabilization was examined with three examples; the first one (Figure 12) shows that the slope is still unstable, the sec-ond one (Figure 13) implies that block remov- al has caused the slope to become stable, and the third one (Figure 14) shows the effects of a berm on the slope and reveals how applying a berm can increase the stability of the slope.

Blocks displacements during several steps in the analyses are shown in Figure 15. Compar- ison reveals that the slope in Figure 14 needs more block removal than the one in Figure 13 to become stable.

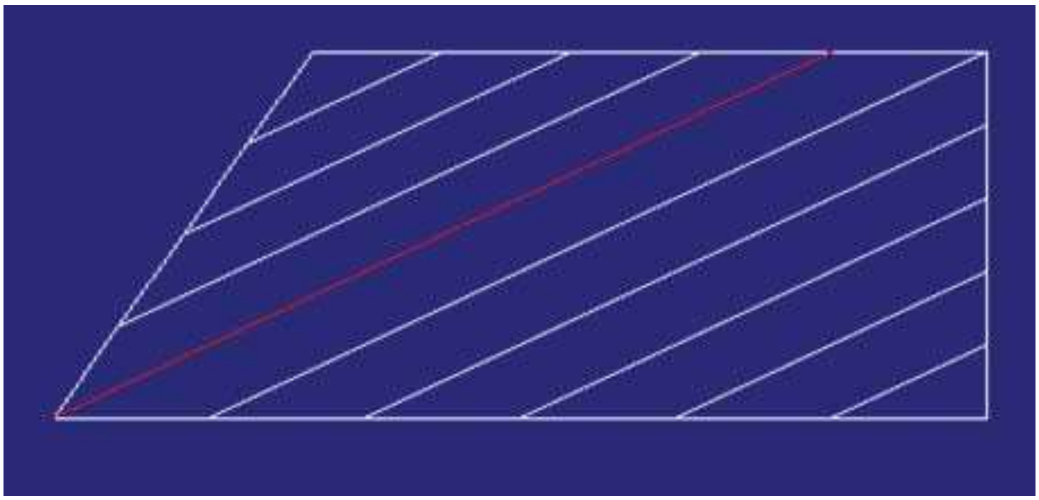

Figure 10. Rock slope model and joints directions (red colour indicates that shear failure has oc-curred at the joint)

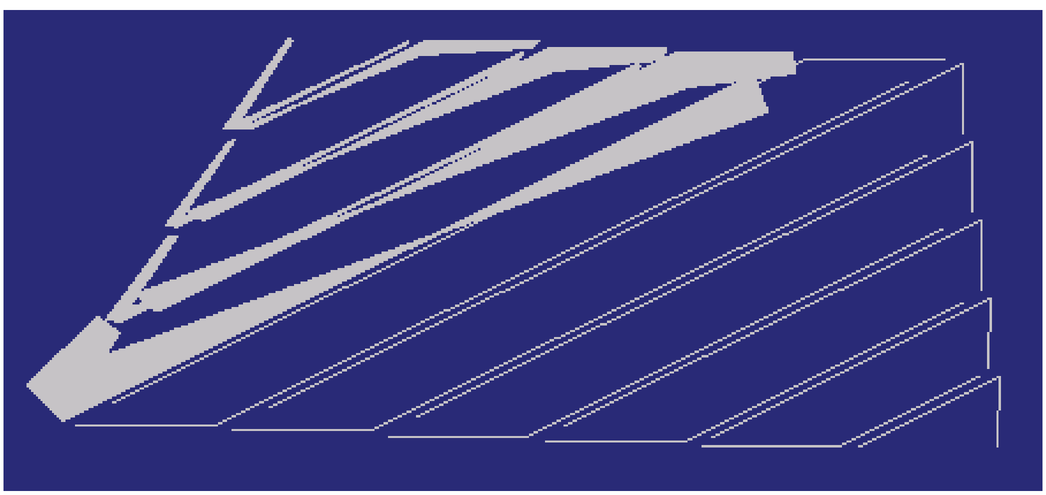

Figure 11. Rock blocks displacements

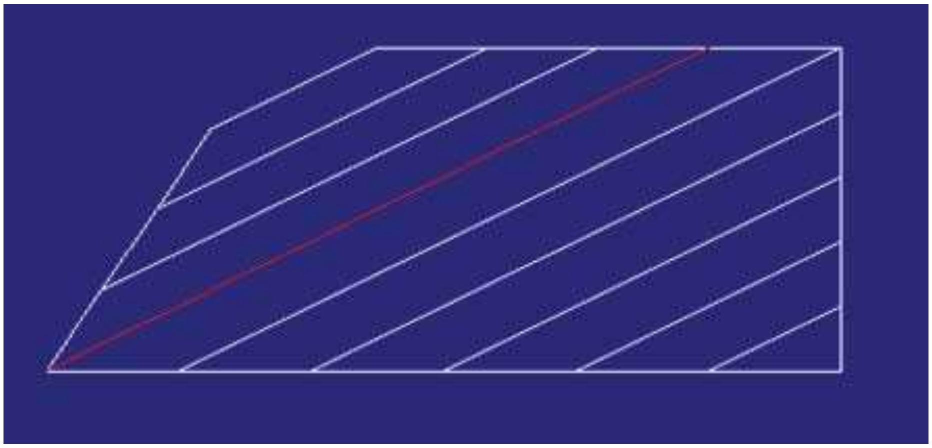

Figure 12. Stability analysis of slope with block removal (the slope is still unstable)

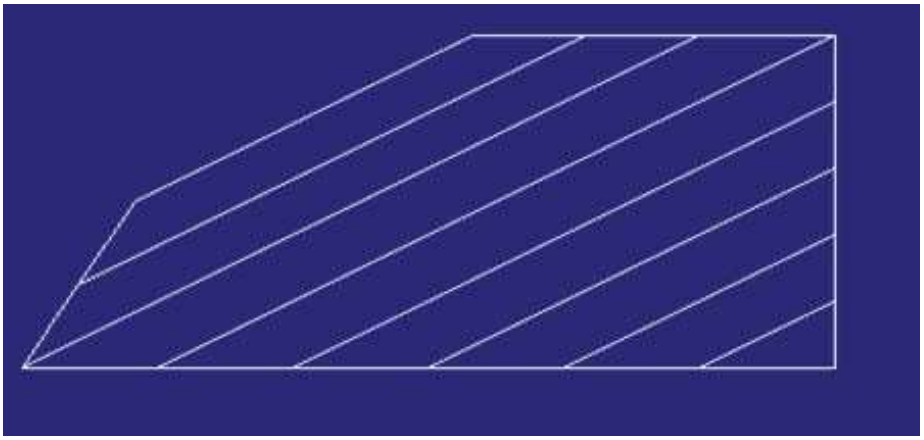

Figure 13. Stability analysis of slope with block removal (the slope is stable)

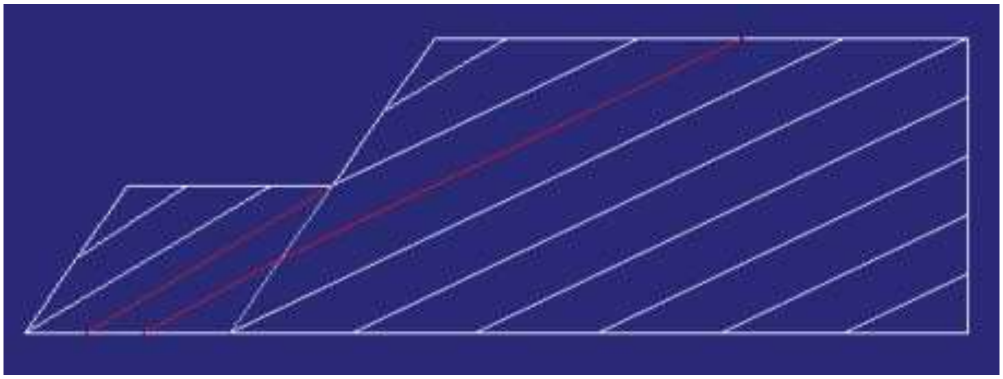

Figure 14. Stability analysis of a bermed slope with inclined layers

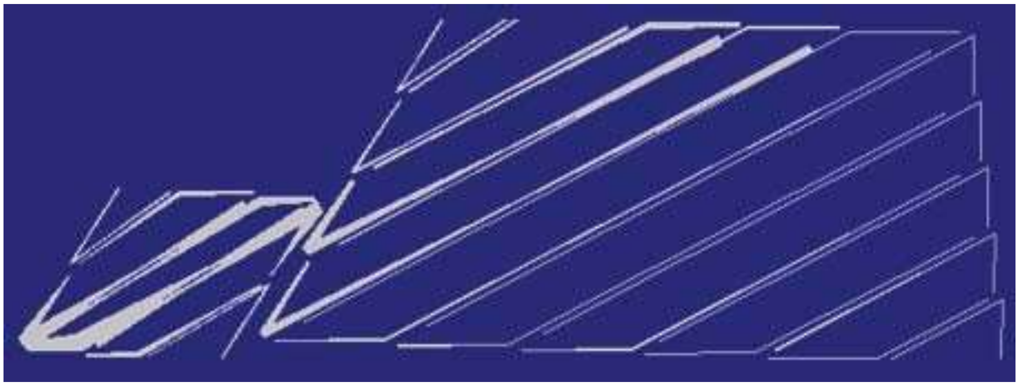

Figure 15. Displacements of rock blocks in a bermed slope with inclined layers

Toppling Mode Stability Analysis

The rock slope for the modelling of the toppling mode is similar to that in Section 6.1, but with near vertical joints. Figure 16 shows the model and joints directions and Figure 17 shows the blocks displacements and indicates that failure has occurred in this example. It is evident from Figure 16 that block removal cannot stabilize the slope because shear fail-ure has occurred in all the joints.

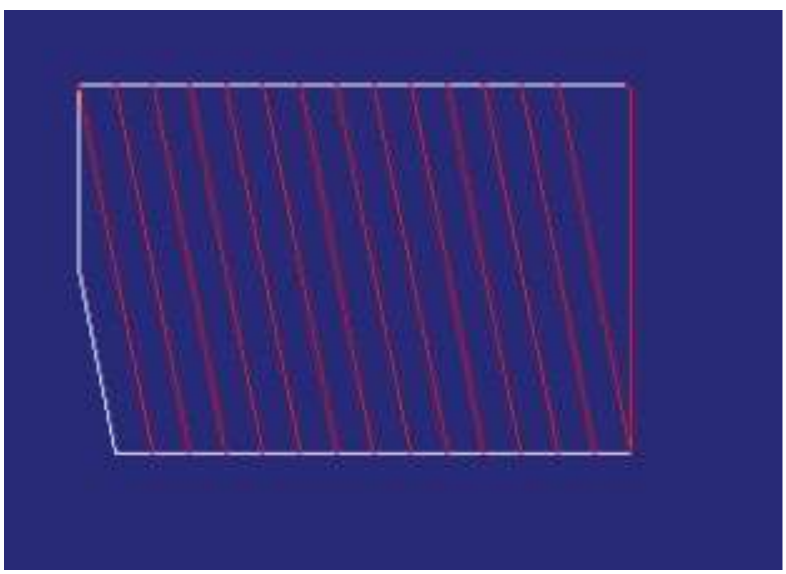

Figure 16. Stability analysis of a toppling mode

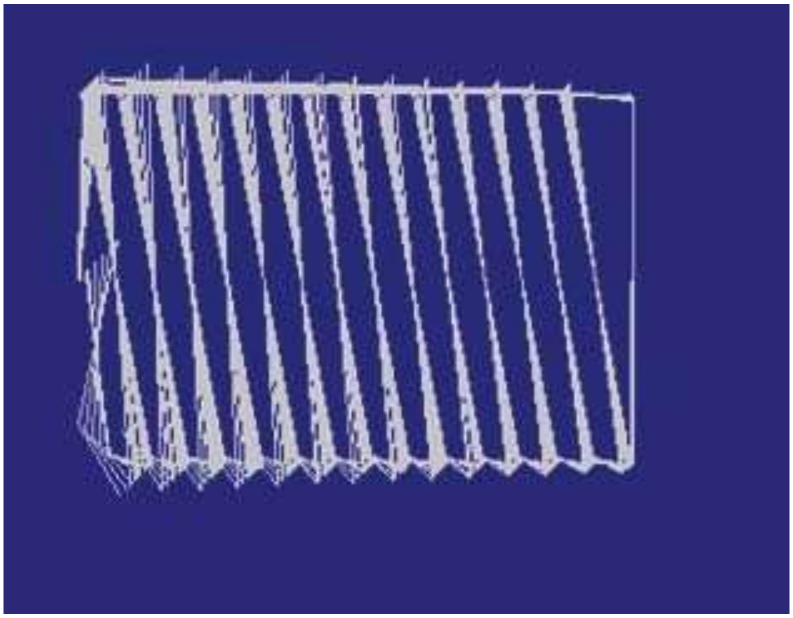

Figure 17. Displacements of rock blocks in the stability analysis of a toppling mode

7. Conclusions

In this study, the Discrete Element Method (DEM) was adopted for the stability analyses of rock slopes. To demonstrate its applicabil-ity and usefulness in analyzing and optimiz- ing the slope stabilization, several examples including the stability analyses of a slope with inclined layers, a slope with two sets of perpen- dicular joints, and a toppling mode have been presented and its advantages over the conven- tional limit equilibrium method has also been discussed. Progressive failure with which the slip surface or unstable blocks can be defined, is a subject considered in this method; there- fore, the method helps in optimizing the rock slope stabilization. The proposed method is theoretically rigorous and simple; it can eas- ily treat such more complicated problems as a slope with inhomogeneous properties and forces acting or foundations resting on a slope.

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