Performance Analysis of Bode's Ideal Loop Transfer Function based Fractional-order Controller for Time Delay systems

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Abstract The phenomenon of time-delay occurs in various real-world, man-made systems and engineering applications such as chemical and biological processes, hydraulic systems, and manufacturing processes etc. The presence of time-delay tends to degrade the performance of the control system. The instabilities due to occurrence of time delay in system adds design complexity for controller design and overall analysis of performance. From the literature survey it is observed that many researchers tried to design traditional and advanced controller to handle this system dynamic. It is also reported that Fractional-order controllers (FOC) are more effective than conventional integer-order (IO) controllers. To ensure a robust closed-loop configuration for time delay systems, Bode's Ideal Loop Transfer Function (BITF) is presented in this paper. The performance of BITF based FOC and modified BITF based FOC for three different class of time delay systems are compared and analysed. From the simulation results it is observed that the performance of BITF based fractional-order controller is more robust than traditional controller.

Keywords Time delay, Bode's Ideal Loop Transfer Function, Integer-order Controller, Fractional-order controller.

I. Introduction

In last few decades, there has been a rise in interest in time-delayed systems from control researchers and engineers. Time delay is a typical aspect in the all branches of engineering. It is well known that time delays are frequently encountered in electrical, electronic, communication systems, control systems, power systems transmission lines and many real world applications. The various industrial processes include after effect phenomena in inner dynamics. Actuators, sensors, and field networks that are involved in feedback loops introduce unavoidable delays. The theoretical research on delay systems is crucial, and is definitely a growing field for researchers. A controller design must be worked upon for necessary application in design process for highlighting good properties of time delays that can bring about new advances practical design [1][2][3][4]. Integer-order Proportional Integral Derivative (PID) controllers are the prominent controllers used in industry for their simplicity, robustness, and availability of effective and easy tuning methods based on minimum plant model knowledge.

In recent years, it is observed that there is a rapid increase in number of studies related to applications of fractional calculus (FC) theory in many areas. The range of applications covers both controller design and synthesis. An exclusive feature expresses real systems better than integer-order ones [5][6]. A delay-dependent robust stability for systems with time-varying delay must be considered. With the increasing expectations of system dynamic performances, the delay information in practical system/process must be observed in the modelling procedure. Time delays often degrade the performance of main system. These delays cause instabilities of original systems and make it difficult for system analysis. In control theory engineering, improvising or optimizing performance plays a major role. Therefore, the aim of the paper is to employ FOC to boost IO dynamic system

control performance for integer-order plants, fractional-order plants and plants with time delay [7][8][9] [10][11].
In this paper, Bode's ideal loop transfer function method is used to design fractional-order controllers. The advantage of this control design method is that the closed loop systems step responses exhibit an iso-damping property [1] [5] [12-22]. Iso-damping is a desirable system property referring to a state, where the open-loop phase Bode plot is flat. For systems that exhibit iso-damping property, the overshoots of the closed-loop step responses will remain almost constant for different values of the controller gain. This will ensure that the closed-loop system is robust to gain variations and to keep phase margin constant, the phase derivative with respect to frequency should be zero around the gain cross-over frequency.
The paper is organized as follows. The modelling of Time delay system is described in section II, followed by FOPID controller Design in section III. Further, section IV describes Bode's ideal loop transfer function based controller and its basic properties is presented. Finally, section V illustrates simulation examples and section VI concludes with general discussion on the results and conclusion.

II. Modelling of Time Delay System

In numerous control systems, the presence of time delay is a source of instability and oscillation creation [22-28]. Systems comprising internal delays represent a class of systems with a very complex dynamics characterized by an infinite spectrum, specific responses and characteristics in time and frequency domains. Tasks of their control often require the use of unusual solutions that are very specific, unlike conventional approaches known for systems with no delay. The presence of delays significantly influences the feedback dynamics; especially, it has a decisive impact to stability that can be sensitive to even small delay value changes. A considerable amount of the existing solutions of time delay controller synthesis is based on highly advanced mathematical operations from the field of matrix calculus or calculus of variations, which (from the engineering point of view) makes their practical applicability more difficult or even impossible [29-35].
The systems can be approximated by an FOPDT model, the different types of Time Delay systems are modelled as;

1. First Order Plus Time Delay (FOPTD) Model
2. Second Order Plus Time Delay (SOPTD) Model

The First Order Plus Time Delay system has the following form of mathematical model:

$$G(s) = \frac{k}{Ts+1} e^{-Ls}, \tag{1}$$

Where, k is the process gain, L and T are the delay and time constant of the system, respectively. The process gain k is assumed to be unity since all the systems are normalized; The FOPDT models are characterized by a very important parameter called the relative dead time of the system, defined as

$$\tau = \frac{L}{L+T}, \tag{2}$$

Parameter τ ranges between 0 and 1. Systems in which L >> T are called "delay dominant" and systems in which T << L are called "lag dominant". In this paper, a lag dominated system is considered.

III. Controller Design for Time Delay System

There are numerous types of IO and FO controllers used in various applications depending on the application specific requirements. The classical control approaches for time-delay systems are summarised as classical proportional‐integral‐derivative (PID) control, adaptive control, sliding mode control, relay control [15-19]. A PID controller consists of three terms/modes/actions: proportional, integral and derivative. Different combinations of these terms result in different controllers, such as PI controllers and PD controllers.

The transfer function representation of the Integer-order PID (IO-PID) controller in parallel structure is given in (Eq3),

$$C(s) = K_p + \frac{K_i}{s} + K_D s, \tag{3}$$

Where, K_p, K_i, K_D are the controller gains.
A control system is often designed to meet specified gain and phase margins so that the system is robustly stable.
The paper focuses on analysis of FOC. The family of FOPID Controllers types as Fractional-order PID controller, CRONE Controllers, Fractional Lead-Lag Compensator. Among them, Fractional-order PID (FOPID) controller is prominently used. The controllers mentioned in (Oustaloup and Melchior 1993) are also used [16] [17] [18].
The corresponding generalized FOPID controller is given in (Eq4),

$$C(s) = K_p + \frac{K_i}{s^{\lambda}} + K_D s^{\mu}, \tag{4}$$

Where, λ, μ are the fractional-order operators. FOPI and FOPD controllers are possible by usage of the fractional integral or derivative terms separately.
For closed-loop control systems, there are four situations: (1) integer order (IO) plant with IO controller; (2) IO plant with fractional-order (FO) controller; (3) FO plant with IO controller, and (4) FO plant with FO controller.
The structure of FOPID controllers with Time Delay is shown in Figure 1.

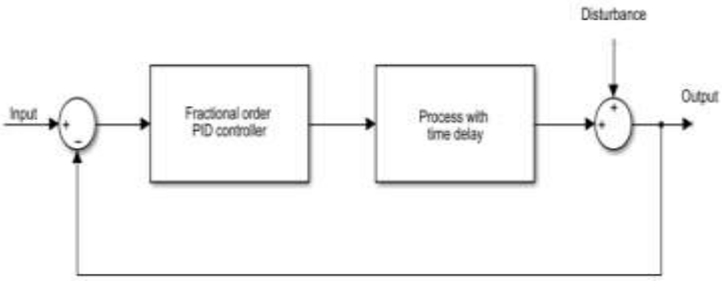

Figure 1 Structure of FOPID controllers with Time Delay System [9]

The conventional tuning algorithms used in the design of fractional-order controller are based on a frequency domain approach, Ziegler-Nichols rule [24], the tuning of FOPIDs using an extension of the popular Ms Constrained Integral Optimization (MIGO) method, named as the F-MIGO (Fractional-MIGO) optimization [34], other tuning methods based on certain performance index [24] based on minimizing some time domain cost functions, such as Integral of Square Error (ISE), Integral of Time Absolute Error (ITAE), Integral of Absolute Error (IAE), etc. are presented [12] [13] [14] [15]. The vast literature shows that different controller exhibit distinctive characteristics and therefore handling dynamics of plants and meeting desired specifications is a tedious task. Therefore controllers with robust structure must be identified with promising applications. Section IV highlights one such method for robust design.

IV. Bode's Ideal Loop Transfer Function based Robust controller

Towards the middle of 20th century, Bode proposed the first idea involving the use of FOC in a feedback problem known as Bode's ideal transfer function. A key problem in the design of a feedback amplifier was to devise a feedback loop so that the performance of the closed-loop is invariant to changes in the amplifier gain [1][3][22][34]. Bode proposed that the ideal shape of the Nyquist plot for the open loop frequency response is a straight line in the complex plane, which provides theoretically infinite gain margin. Bode emerged with the first sign of the potential of FOC, though without using the term fractional-order. Bode presented an elegant solution to this robust design problem.

A fractional-order integrator is given in (Eq5)

$$L(s) = \frac{{\omega_{gc}}^{\alpha}}{s}, \tag{5}$$

is known as Bode's ideal transfer function, where ω_{gc} is the gain crossover frequency and the constant phase margin is $\Phi_m = \pi - \frac{\alpha\pi}{2}$. The Bode diagram of $L(s)$ ($1 < \alpha < 2$) is shown in Figure 2. This frequency characteristic is interesting in terms of robust feature of the system to parameter changes or uncertainties, and many researchers have used this design method. The major benefit achieved through this structure is iso-damping, i.e. overshoot being independent of the system gain. The use of fractional elements for description of ideal Bode's control loop is a promising application of FC in the process control field [1][2].

Bode's ideal control loop frequency response has the fractional integrator shape and provides the iso-damping property around the gain crossover frequency. This is due to the fact that the phase margin and the maximum overshoot are defined by one parameter only (the fractional power of α), and are independent of open-loop gain. Bode's ideal loop transfer function is probably the first design method that addressed robustness explicitly. The design of FO controllers using Bode's ideal transfer function [4] [5] is one of the applications. The best FOC can outperform the most accurate IO controllers are documented in literature [22].

In this paper, an approach for design of fractional-order controllers using Bode's ideal loop transfer function for IO, FO plants and time delay plants is presented. The advantage of this control design methodology is that the closed loop systems step responses exhibit an iso-damping property. Iso-damping is a desirable system property referring to a state, where the open-loop phase Bode plot is flat, i.e. the phase derivative with respect to frequency is zero, at a frequency called the tangent frequency. At the tangent frequency the Nyquist curve of the open-loop system tangentially touches the sensitivity circle and the phase Bode is locally flat which implies that the system will be more robust to gain variations. For systems that exhibit iso-damping property, the overshoots of the closed-loop step responses will remain almost constant for different values of the controller gain. This will ensure that the closed-loop system is robust to gain variations and to keep phase margin constant, the phase derivative with respect to frequency should be zero around the gain cross-over frequency.

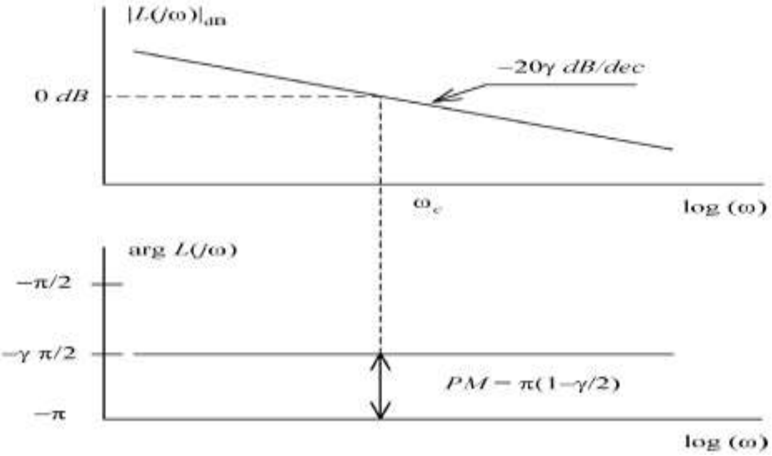

Figure 2 Bode diagrams of amplitude and phase of $L(s)$ for $1 < \gamma < 2$ [Barbosa]

A new Bode's ideal transfer function is designed to tolerate the time-delay in loop is proposed in [9]. A time delay Bode's model for FOPID design is selected. The choice of $G(s)$ as open-loop transfer function also gives an ideal closed-loop model under a unit feedback with infinite gain margin and constant phase margin

$$G(s) = \frac{\omega_c^{\alpha}}{s^{\alpha} + \omega_c^{\alpha}}, \tag{6}$$

It is well known that, the bandwidth design for time-delay system is a critical problem in control system design. However, this problem is rarely discussed in the current Bode shaping methods [34][35]. This problem is solved by investigating the gain and phase margins for the proposed time-delay Bode's model. Bode shaping for FOPID design is solved by one dimensional searching, rather than five parameter optimizations. To simplify the design of FOPID from solving nonlinear equations, data fitting at steady state and the cross over frequency are derived, such that five unknown parameters are reduced to one [9]. Then, the problem is easily solved by one dimensional searching. The basic idea is shown in block diagram of the FOPID control system in Figure 2.
The following time delay system is used,

$$P_1(s) = P_2(s)e^{-Ts}, \tag{7}$$

Where, T is time delay, $P_1(s)$ is the controlled plant and $P_2(s)$ is delay free model. The closed loop system with a FOPID controller is given in (Eq8)

$$\frac{P_1(s)C(s)}{1 + P_1(s)C(s)} = \frac{P_2(s)C(s)}{1 + P_2(s)e^{-Ts}C(s)}e^{-Ts}. \tag{8}$$

It is observed that time delay exists in closed loop system. The BITF and time delay can be combined and a desired model of closed loop for time delay system can be taken as

$$H(s) = \frac{\omega_c^{\alpha}}{s^{\alpha} + \omega_c^{\alpha}}e^{-Ts}, \tag{9}$$

Where, time delay in $H(s)$ equals to real one.

V. Design and Simulation Examples

Example1. A generalized unstable integer-order open-loop transfer function is,

$$P(s) = \frac{(s+1)(s+2)}{(s+0.1)(s-1)} \tag{10}$$

Design specifications:

- Phase margin (PM)= 100˚
- Gain crossover frequency = 1000 rad/s

Solution:

1. The transfer function being proper in nature and can be used directly for design of FO controller.
2. FO controller is designed to obtain the loop transfer function as Bode's Ideal Integrator Eq5.
3. Here we want PM of 100˚ at 1000rad/sec. Thus using equation Bode's equation, we get k_c = 464.1945 and α= 0.8889.
4. Thus fractional-order controller obtained is,

$$C(s) = \frac{464.1945(s+1)(s+2)}{s^{0.8889}(s+0.1)(s-1)} \tag{11}$$

5. The implementation of FO controller is done by Oustaloup's Recursive Approximation (ORA) which approximates an FO operator as a chain of first-order filters within a specified frequency band (ω_l , ω_h) as (100,1000000). Here the Modified Oustaloup's Recursive Approximation is used.

The Bode plot in Figure 3 of the forward path transfer function C(s)P(s) shows that the designed FO controller has achieved the desired closed-loop phase margin of 100˚. The plot also contains the frequency response of $C(s)P(s) = \frac{k_c}{s^{\alpha}}$. It is seen that these overlap each other in the given frequency range. The figure 4 shows step response of the plant P(s) with controller. The designed FO controller is compared with Proportional Integral Derivative (PID) controller using PID toolbox of

MATLAB. The Integral controller with Kp= 0.25629, Ki=0.04032, Kd=0.40725 is obtained which shows that we can tune a PID controller.

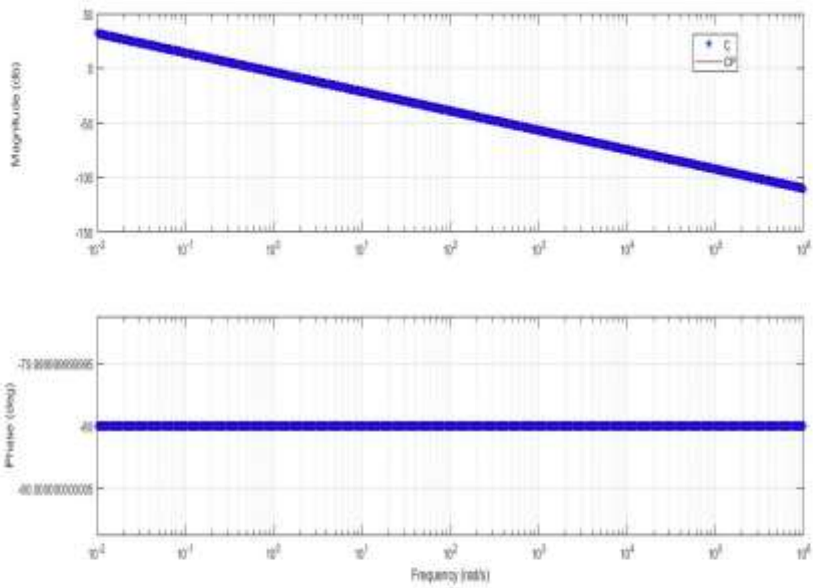

Figure 3 Bode plots with designed FO controller for $P(s)$ for Example1

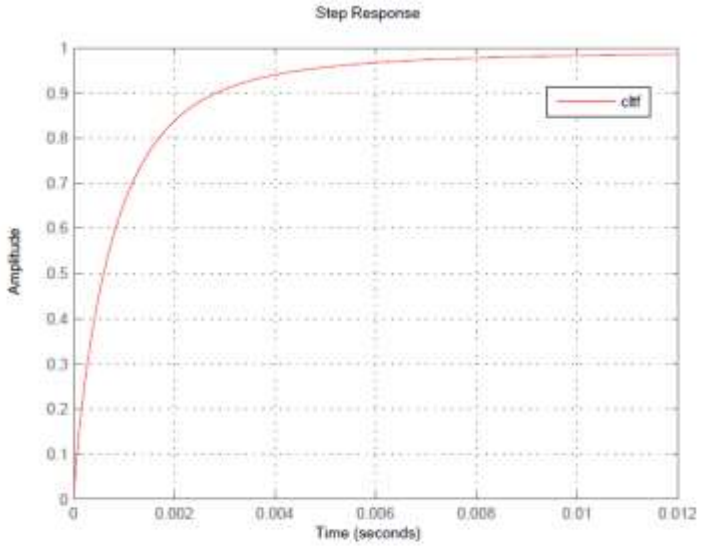

Figure 4 Step response with designed fractional-order controller for Example1

From the table 1 it can be clearly seen that the designed FO controller gives an excellent performance with a less rise time and faster settling time as compared to PID controller. The overshoot has also reduced with the designed fractional-order controller as compared to PID controller.

Table 1 Time domain parameter specification for Example1

Parameters	**Without controller**	**With PID controller**	**With BITF Fractional-order Controller**
Rise time	-	1.05 seconds	0.003 seconds

Settling time	-	82.5 seconds	0.0128 seconds
Overshoot	6.24×10^{26}	77.6	0.986

Example2. A generalized fractional-order open-loop transfer function is,

$$P(s) = \frac{1}{(s^{0.2} + 1)} \tag{12}$$

Design specifications:

- Phase margin (PM)= 100˚
- Gain crossover frequency = 1000 rad/s

Solution:

1. The transfer function being proper in nature, so it can be used directly for design of FO controller.
2. FO controller is designed to obtain the loop transfer function as Bode's Ideal Integrator from (Eq5).
3. Here we want PM of 100˚ at 1000rad/sec. Thus using equation (5) we get k_c = 464.2 and α= 0.89.
4. Thus fractional-order controller obtained is,

$$C(s) = \frac{464.2}{s^{0.89}\, P(s)} \tag{13}$$

5. The implementation of FO controller is done by Oustaloup's Recursive Approximation(ORA) which approximates an FO operator as a chain of first-order filters within a specified frequency band (ω_l , ω_h) as (100,1000000). Here the Modified Oustaloup's Recursive Approximation is used.

The Bode plot in Figure 5 of the forward path transfer function C(s)P(s) shows that the designed FO controller has achieved the desired closed-loop phase margin of 100˚. The plot also contains the frequency response of $C(s)P(s) = \frac{k_c}{s^{\alpha}}$. It is seen that these overlap each other in the given frequency range. Figure 6 shows step response of the plant P(s) with controller. The table shows the parameters specification with and without controller and it is clear that the designed FO controller stabilizes the plant faster at 0.223 seconds.

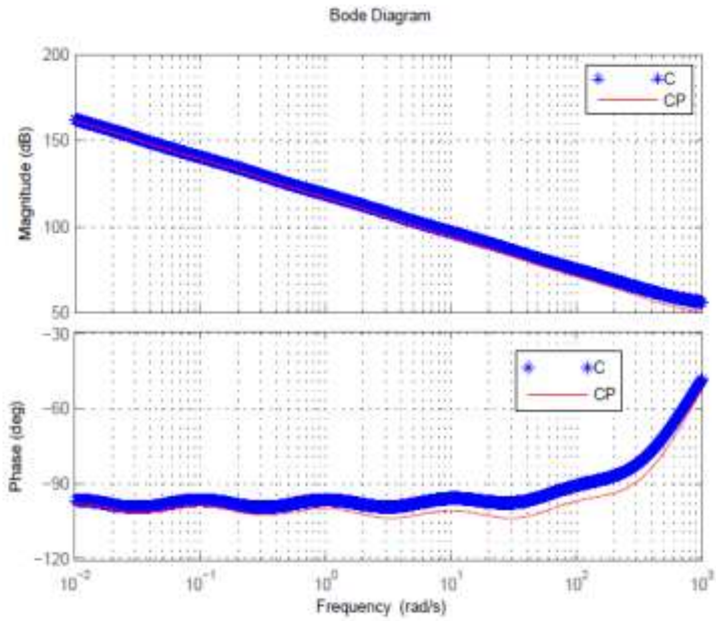

Figure 5 Bode plots with designed fractional-order controller for Example2

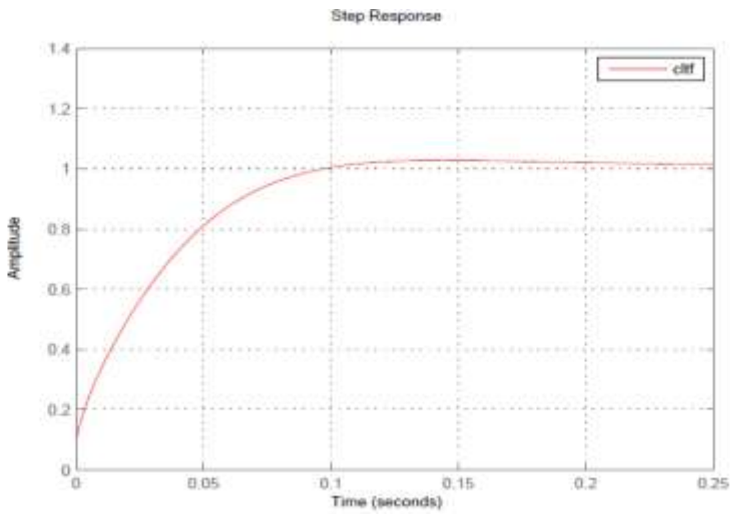

Figure 6 Step responses with designed fractional-order controller for Example2

Table 2 Time domain parameter specification for Example2

Parameters	**Without controller**	**With BITF Fractional-order Controller**
Rise time	693 seconds	0.0693 seconds
Settling time	-	0.223 seconds
Overshoot	2.22×10^{-14}	2.83
Steady state value	0.994	1

Example3. A FOPTD system of literature [10] is considered,

$$G(s) = \frac{1}{s+1} e^{-0.1s} \tag{14}$$

This is a lag-dominant FOPDT with K=1, L=0.1 and T=1. The time constant is much larger than the delay. The plant dynamics are shown in Table 3

Table 3 FOPTD plant $G(s)$ frequency and time domain parameters

Controllers	**Gain Margin**	**Phase Margin(deg)**	**Gain cross over frequency(rad/s)**	**Overshoot (%)**	**Rise time(s)**	**Settling time(s)**
FOPTD plant $G(s)$	16.3187	-180	0	0	2.1971	4.0121

The design procedure [9] is summarized as below:

1. The parameters ω_{gc} and α are found as 4.85 and 1.01 by taking into account the stability constraints.
2. The gain margin (GM) is estimated at 3.2.
3. The phase margin is determined as 61˚.
4. A simple and effective FOPID controller designed in frequency domain is determined with the differential order μ as 0.68 and the final FOPID controller is obtained [9] as

$$C_{FOPID}(s) = 3.1534 + \frac{4.9272}{s^{1.01}} + 0.1487s^{0.68} \quad (15)$$

A fractional-order FOPI controller is designed by Luo [33] as

$$C_{FOPI}(s) = 3.3367 + \frac{4.6464}{s^{1.21}} \quad (16)$$

An integer-order PI controller is optimized by Astrom and Hagglund [17] for same $G(s)$ FOPTD system as

$$C_{FOPI}(s) = 2.8236 + \frac{4.6464}{s} \quad (17)$$

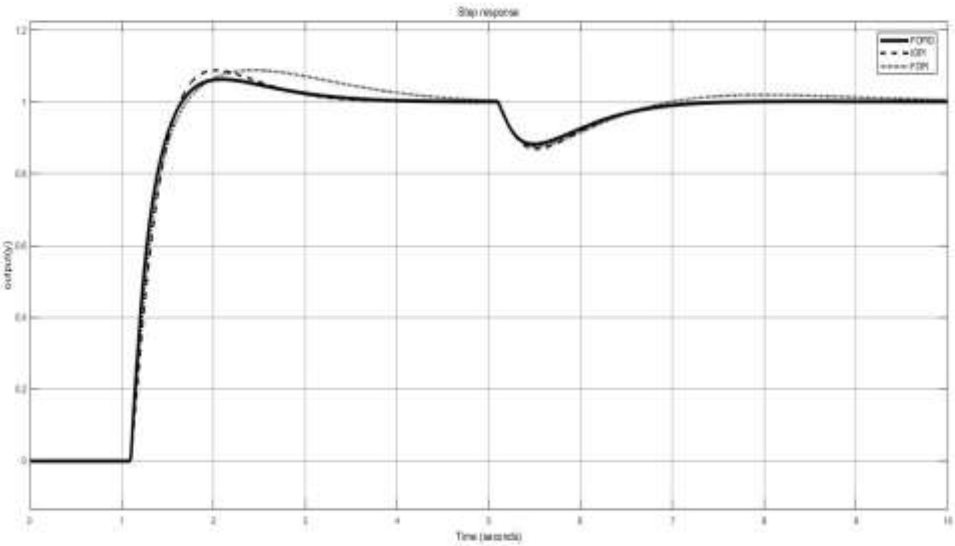

Figure 7 Step responses with $C_{FOPID}(s), C_{FOPI}(s), C_{IOPI}(s)$ controllers

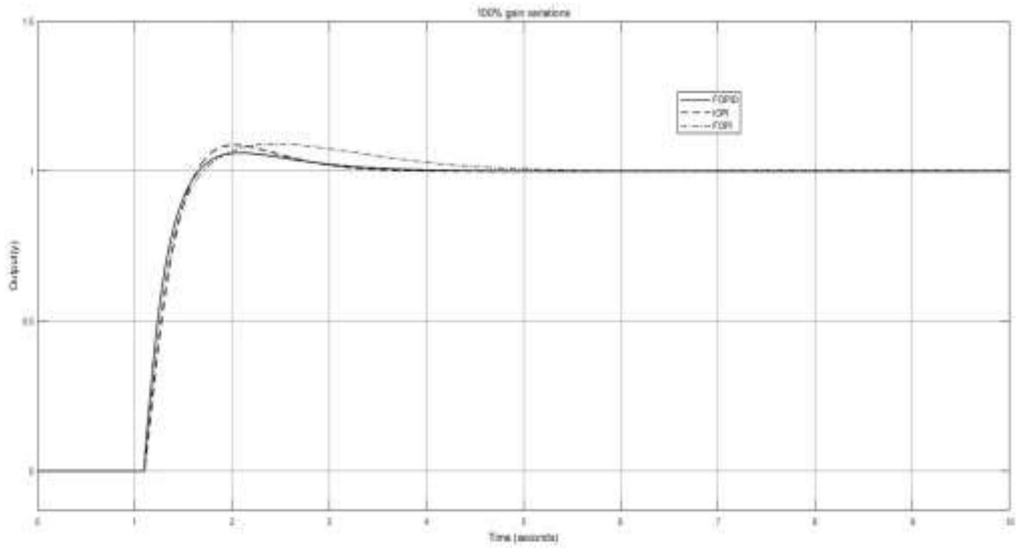

Figure 8 Step responses with +10% gain variations for $C_{FOPID}(s), C_{FOPI}(s), C_{IOPI}(s)$ controllers

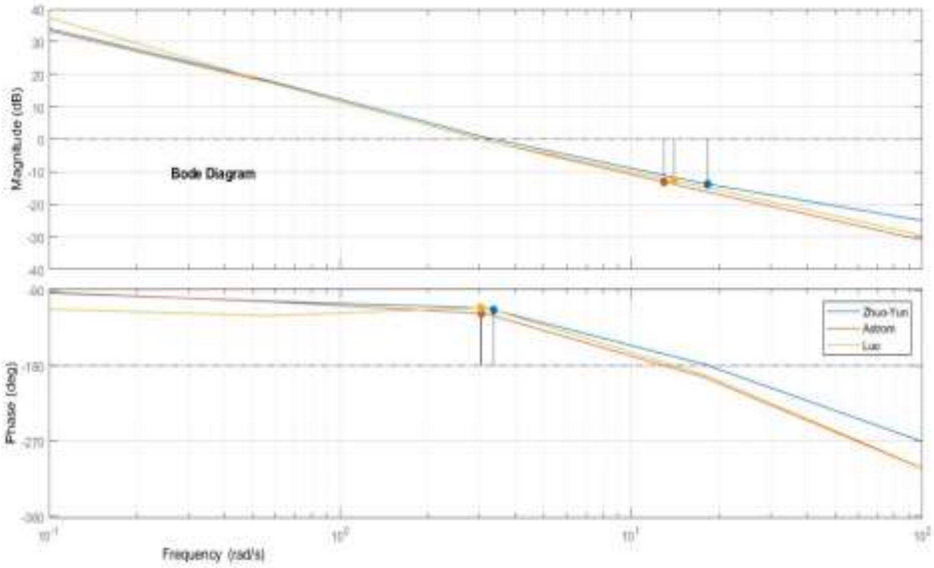

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Figure 9 Bode plots with $C_{FOPID}(s), C_{FOPI}(s), C_{IOPI}(s)$ controllers

VI Result Analysis

To illustrate the set-point tracking and disturbance rejection performance, step response and load disturbance response are presented in Figure 7. It is clearly seen that the FOPID in (Eq15) provides better control performance than the controllers in equation (Eq16) and (Eq17), which is also demonstrated by their frequency response of open-loop transfer functions in Figure 9. The comparison indexes [9] gain margin, phase margin, gain cross over frequency, overshoot, rise time and settling time for the set point response shows the effectiveness of the method.

To compare the robustness of three controllers, the step responses with +10% gain variations is also shown in Figure 8. The performance shows the system robustness to the gain uncertainties using the FOPID controller. Compared with the step responses using the FOPI and IOPI controllers, the overshoots of the FOPID are smaller and with shorter settling time.

VII Conclusions

This paper present successful design and implementation of robust BITF based fractional-order and modified BITF based fractional-order controller for generalized unstable integer-order time-delay system, generalised fractional-order time delay system and FOPDT system. The closed loop performance parameters like gain margin, phase margin, gain cross over frequency, overshoot, rise time and settling time for the set point response shows the effectiveness of the proposed method. The performance of proposed controllers is also tested with parametric uncertainty. Simulation results shows improvement in robust behaviour of designed controller for time delay systems. Research survey indicates design of soft computing and nature inspired based robust controller for such time-delay system.

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