Transverse Momentum Distributions in High-Energy Physics under Relativistic Thermodynamics.

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Abstract.
The Tsallis distribution can be used to analyse the transverse momentum distributions measured by the STAR and PHENIX collaborations at the Relativistic Heavy Ion Collider and by the ALICE [3], ATLAS [4], and CMS [5] collaborations at the Large Hadron Collider in the context of relativistic thermodynamics. Thermodynamic consistency in the context of relativistic high energy quantum distributions is theoretically elucidated. The transverse momentum distribution is described in an improved form, and fits are shown together with estimates for the parameter q and the temperature T.

1 Introduction

Over the past few years, a vast amount of fresh data has been generated by the Large Hadron Collider (LHC) and the Relativistic Heavy Ion Collider (RHIC). Due to this, a new energy zone is now accessible for relativistic thermodynamics and hydrodynamics can be tested and applied. The highest available energy for heavy ions is $\sqrt{s} = 2760$ AGeV yet the observed temperature is only of the order of $T \approx 0.16$ GeV at RHIC as well as at the LHC. This enormous change from the energy available in the initial state to the temperature observed in the final state is clearly a challenge for dynamical models.

In the analysis of the new data, one statistical distribution has gained prominence with very good fits to the transverse momentum distributions made by the STAR [1] and PHENIX [2] collaborations at RHIC and by the ALICE [3], ATLAS [4] and CMS [5] collaborations at the LHC.

In the literature there exists more than one version of the Tsallis distribution [6, 7, 8, 9, 10] and we investigate here one that we consider well suited for describing results in high energy physics. Our main guiding criterium will be thermodynamic consistency which has not always been implemented correctly (see e.g. [11, 12, 13]). The explicit form which we will use is [14]:

$$\frac{d^2N}{dp_T\,dy} = gV\frac{p_T m_T \cosh y}{(2\pi)^2}\left[1 + (q-1)\frac{m_T \cosh y - \mu}{T}\right]^{-q/(q-1)}, \tag{1}$$

where p_T and m_T are the transverse momentum and mass respectively, y is the rapidity, T and μ are the temperature and the chemical potential, V is the volume, g is the degeneracy factor. In the limit where the parameter q goes to 1 this reduces the standard Boltzmann distributio

$$\lim_{q\to1}\frac{d^2N}{dp_T\,dy} = gV\frac{p_T m_T \cosh y}{(2\pi)^2}\exp\left(-\frac{m_T \cosh y - \mu}{T}\right).$$

The parameterization given in Eq. (1) is close to the one used by the STAR, PHENIX, ALICE, ATLAS and CMS collaborations [1, 2, 3, 4, 5]:

$$\frac{d^2N}{dp_T\,dy} = p_T\frac{dN}{dy}\frac{(n-1)(n-2)}{nC(nC + m_0(n-2))}\left[1 + \frac{m_T - m_0}{nC}\right]^{-n} \tag{3}$$

where n, C and m_0 are fit parameters. The analytic expression used in Refs. [1, 2, 3, 4, 5] corresponds to identifying

$$n \to \frac{q}{q-1} \tag{4}$$

and

$$nC \to \frac{T + m_0(q-1)}{q-1}. \tag{5}$$

After this substitution Eq. (3) becomes

$$\frac{d^2N}{dp_T\,dy} = p_T\frac{dN}{dy}\frac{(n-1)(n-2)}{nC(nC + m_0(n-2))}\left[\frac{T}{T + m_0(q-1)}\right]^{-q/(q-1)}\left[1 + (q-1)\frac{m_T}{T}\right]^{-q/(q-1)}. \tag{6}$$

Which, at mid-rapidity $y = 0$ and zero chemical potential, has the same dependence on the transverse momentum as (1) apart from an additional factor m_T . The inclusions of the factor m_T leads to a more consistent interpretation of the variables q and T . In particular, no clear pattern emerges for the values of n and C while an interesting regularity is obtained for q and T as seen in Figs. 8 and 9 shown towards the end of this paper.
The Tsallis distribution introduces a new parameter q which in practice is always close to 1, typical values for the parameter q obtained from fits to the transverse momentum distribution are in the range 1.1 to 1.2, in the remainder of this paper we will always assume $q > 1$.

In section 2 we review the derivation of the Tsallis distribution by emphasizing the quantum statistical form. In section 3 we prove thermodynamic consistency. In section 4 we show in detail fits to the transverse momentum distribution in $p - p$ collisions at $\sqrt{s}$ = 900 GeV. Section 5 presents conclusions.

2 Tsallis Distribution.

Comparison with Standard Statistical Distributions

The Tsallis form of the Fermi-Dirac distribution proposed in [13,15,16,17,18] uses

$$f_T^{FD}(E) \equiv \frac{1}{\exp_q\left(\frac{E-\mu}{T}\right)+1}. \qquad (7)$$

where the function $\exp_q(x)$ is defined as

$$\exp_q(x) \equiv \begin{cases} [1+(q-1)x]^{1/(q-1)} & \text{if} \quad x > 0 \\ [1+(1-q)x]^{1/(1-q)} & \text{if} \quad x \leq 0 \end{cases} \qquad (8)$$

and, in the limit where $q \to 1$ reduces to the standard exponential:

$$\lim_{q\to 1} \exp_q(x) \quad \exp(x).$$

The form given in Eq. (7) will be referred to as the Tsallis-FD distribution. A comparison between the standard Fermi-Dirac and Tsallis-FD distributions as a function of the energy E is shown in Fig. 1 for various values of the temperature T. The Tsallis parameter q is kept fixed at $q = 1.1$. The Bose-Einstein version will be referred to as the Tsallis-BE distribution [19]

$$f_T^{BE}(E) \equiv \frac{1}{\exp_q\left(\frac{E-\mu}{T}\right)-1}. \qquad (9)$$

A comparison between the standard Bose-Einstein and Tsallis-BE distributions as a function of the energy E is shown in Fig. 2 for various values of the temperature T . The Tsallis parameter q is again kept fixed at $q = 1.1$. The classical limit will be referred to as Tsallis-B distribution (the B stands for the fact that it reduces to the Boltzmann distribution in the limit where $q \to 1$) and is given by [6, 7]

$$f_T^{B}(E) \equiv \exp_q\left(-\frac{E-\mu}{T}\right). \qquad (10)$$

or, using standard notation,

$$f_T^{B}(E) = \left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{1}{q-1}}. \qquad (11)$$

Again, a comparison between the standard Boltzmann and Tsallis-B distributions as a function of the energy E is shown in Fig. 3. for various values of the temperature T . As before the Tsallis parameter q is kept fixed at $q = 1.1$. All forms of the Tsallis distribution introduce a new pa-

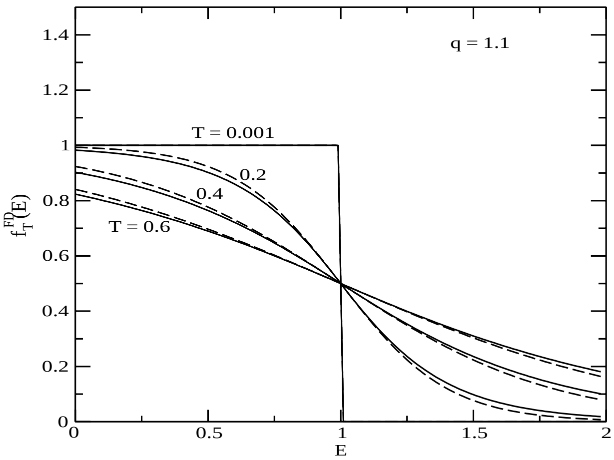

Fig. 1. Comparison between the Fermi-Dirac (dashed line) and the Tsallis-FD (solid line) distributions as function of the energy E, keeping the Tsallis parameter q fixed at 1.1, for various values of the temperature T . The chemical potential is kept equal to one in all curves, the units are arbitrary.

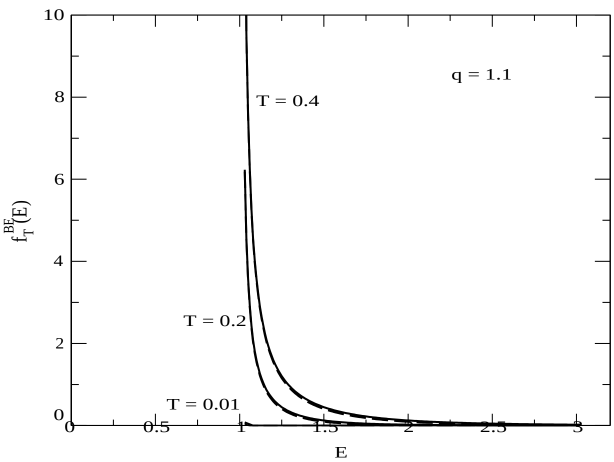

Fig. 2. Comparison between the Bose-Einstein (dashed line) and Tsallis-BE (solid line) distributions as a function of the energy E, keeping the Tsallis parameter q fixed at 1.1, for various values of the temperature T . The chemical potential is kept equal to one in all curves, the units are arbitrary.

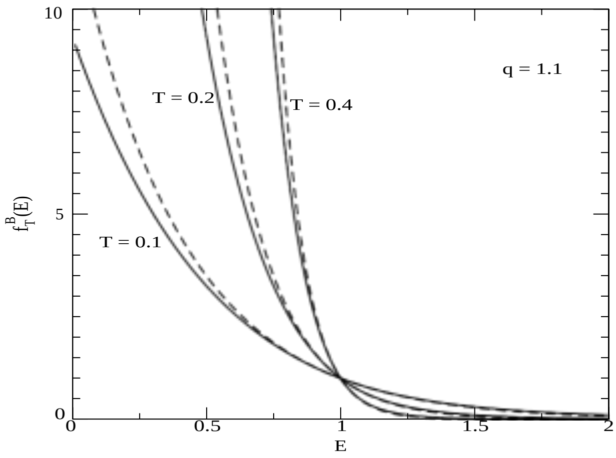

Fig. 3. Comparison between the Boltzmann (dashed line) and Tsallis-B (solid line) distributions as a function of the energy E, keeping the Tsallis parameter q fixed at 1.1, for various values of the temperature T. The chemical potential is kept equal to one in all curves, the units are arbitrary.

rameter q. In practice this parameter is always close to 1, e.g. in the results obtained by the ALICE and CMS collaborations typical values for the parameter q can be obtained from fits to the transverse momentum distribution for identified charged particles [3] and are in the range 1.1 to 1.2 (see below). The value of q should thus be considered as never being far from 1, deviating from it by 20% at most. An analysis of the composition of final state particles leads to a similar result [20] for the parameter q.

In the limit where $q \to 1$ all distributions coincide with the standard statistical distributions:

$$\lim_{q\to 1} f_T^B(E) = f^B(E), \qquad (12)$$

$$\lim_{q\to 1} f_T^{FD}(E) = f^{FD}(E), \qquad (13)$$

$$\lim_{q\to 1} f_T^{BE}(E) = f^{BE}(E). \qquad (14)$$

A derivation of the Tsallis distribution, based on the Boltzmann equation, has been given in Ref. [23,24].

The Tsallis-B distribution is always larger than the Boltzmann one if $q > 1$. Taking into account the large p_T results for particle production we will only consider this case here. As a consequence, in order to keep the particle yields the same, the Tsallis distribution always leads to smaller values of the freeze-out temperature for the same set of particle yields [20].

Derivation for Quantum Statistics

The standard form of the entropy for fermions in statistical mechanics is given in the large volume limit by:

$$S^{FD} = -gV \int \frac{d^3p}{(2\pi)^3} \left[f^{FD} \ln f^{FD} + (1 - f^{FD}) \ln(1 - f^{FD}) \right], \qquad (15)$$

For simplicity Eq. (15) refers to one particle species but can be easily generalized to many. In the limit where momenta are quantized this is given by:

$$S^{FD} = -g \sum_i \left[f_i \ln f_i + (1 - f_i) \ln(1 - f_i) \right], \qquad (16)$$

For convenience we will work with the discrete form in the rest of this section. The large volume limit can be recovered with the standard replacement:

$$\sum_i \to V \int \frac{d^3p}{(2\pi)^3} \qquad (17)$$

The generalization, using the Tsallis prescription, leads to [15,16,17]

$$S_T^{FD} = -g \sum_i \left[f_i^q \ln_q f_i + (1 - f_i)^q \ln_q(1 - f_i) \right], \qquad (18)$$

where use has been made of the function

$$\ln_q(x) \equiv \frac{x^{1-q} - 1}{1 - q}, \qquad (19)$$

often referred to as q-logarithm. The classical limit of this form is given by [21]:

$$S_T^B = -g \sum_i \left[f_i^q \ln_q f_i - f_i \right], \qquad (20)$$

The equilibrium distributions can also be derived from the Rényi distribution as shown in detail in [22]. It can be easily shown that in the limit where the Tsallis parameter q tends to 1 one has:

$$\lim_{q\to 1} \ln_q(x) = \ln(x). \qquad (21)$$

In a similar vein, the generalized form of the entropy for bosons is given by

$$S_T^{BE} = -g \sum_i \left[f_i^q \ln_q f_i - (1 + f_i)^q \ln_q(1 + f_i) \right], \qquad (22)$$

In the limit $q \to 1$ Eqs. (18) and (22) reduce to the standard Fermi-Dirac and Bose-Einstein distributions. Further, as we shall presently explain, the formulation of a variational principle in terms of the above equations allows to prove the validity of the general relations of thermodynamics. One of the relevant constraints is given by the average number of particles,

$$\sum_i f_i^q = N. \qquad (23)$$

Likewise, the energy of the system gives a constraint,

$$\sum_i f_i^q E_i = E. \qquad (24)$$

It is necessary to have the power q on the left-hand side as no thermodynamic consistency would be achieved without it. The maximization of the entropy measure under the constraints Eqs. (23) and (24) leads to the variational equation:

$$\frac{\delta}{\delta f_i}\left[S_T^{FD} + \alpha\left(N - \sum_i f_i^q\right) + \beta\left(E - \sum_i f_i^q E_i\right)\right] = 0, \tag{25}$$

where α and β are Lagrange multipliers associated, respectively, with the total number of particles and the total energy. Differentiating each expression in Eq. (25) separately gives the following results

$$\frac{\delta}{\delta f_i} S_T^{FD} = \frac{q}{q-1}\left[\left(\frac{1-f_i}{f_i}\right)^{q-1} - 1\right] f_i^{q-1}, \tag{26}$$

$$\frac{\delta}{\delta f_i}\left(N - \sum_i f_i^q\right) = -q f_i^{q-1}, \tag{27}$$

and

$$\frac{\delta}{\delta f_i}\left(E - \sum_i f_i^q E_i\right) = -qE_i f_i^{q-1}. \tag{28}$$

By substituting Eqs. (26), (27) and (28) into Eq. (25), we obtain

$$q f_i^{q-1}\left\{\frac{1}{q-1}\left[-1 + \left(\frac{1-f_i}{f_i}\right)^{q-1}\right] - \beta E_i - \alpha\right\} = 0. \tag{29}$$

Which can be rewritten as

$$\frac{1}{q-1}\left[-1 + \left(\frac{1-f_i}{f_i}\right)^{q-1}\right] = \beta E_i + \alpha, \tag{30}$$

and, by rearranging Eq. (30), we get

$$\frac{1-f_i}{f_i} = [1 + (q-1)(\beta E_i + \alpha)]^{\frac{1}{q-1}},$$

which gives the Tsallis-FD form referred to earlier in this paper as [15,16,17]

$$f_i = \frac{1}{[1 + (q-1)(\beta E_i + \alpha)]^{\frac{1}{q-1}} + 1}, = \frac{1}{\exp_q(\alpha + \beta E_i) + 1}. \tag{31}$$

Using a similar approach one can also determine the Tsallis-BE distribution by starting from the extremum of the entropy subject to the same two conditions:

$$\frac{\delta}{\delta f_i}\left[S_T^{BE} + \alpha\left(N - \sum_i f_i^q\right) + \beta\left(E - \sum_i f_i^q E_i\right)\right] = 0, \tag{32}$$

which leads to

$$f_i = \frac{1}{[1 + (q-1)(\beta E_i + \alpha)]^{\frac{1}{q-1}} - 1}, = \frac{1}{\exp_q((E_i - \mu)/T) - 1}. \tag{33}$$

where the usual identifications $\alpha = -\mu/T$ and $\beta = 1/T$ have been made.

Thermodynamic Consistency

The first and second laws of thermodynamics lead to the following two differential relations [25]

$$d\epsilon = Tds + \mu dn, \tag{34}$$

$$dP = sdT + nd\mu. \tag{35}$$

where $\epsilon = E/V$, $s = S/V$ and $n = N/V$ are the energy, entropy and particle densities respectively. Thermodynamic consistency requires that the following relations be satisfied

$$T = \left.\frac{\partial \epsilon}{\partial s}\right|_n, \tag{36}$$

$$\mu = \left.\frac{\partial \epsilon}{\partial n}\right|_s, \tag{37}$$

$$n = \left.\frac{\partial P}{\partial \mu}\right|_T, \tag{38}$$

$$s = \left.\frac{\partial P}{\partial T}\right|_\mu. \tag{39}$$

The pressure, energy density and entropy density are all given by corresponding integrals over Tsallis distributions and the derivatives have to reproduce the corresponding physical quantities, e.g. for Tsallis-B one has

$$n_T^B = g\int \frac{d^3p}{(2\pi)^3}\left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}, \tag{40}$$

$$\epsilon_T^B = g\int \frac{d^3p}{(2\pi)^3} E\left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}, \tag{41}$$

$$P_T^B = g\int \frac{d^3p}{(2\pi)^3}\frac{p^2}{3E}\left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}. \tag{42}$$

For consistency, these expressions have to agree with the basic thermodynamic relations (36), (37),(38), and (39). i.e. , for the above relations, it has to be shown that

$$n_T^B = \frac{\partial P_T^B}{\partial \mu} \tag{43}$$

We prove that this is indeed the case. We will show that the consistency conditions given above are indeed obeyed.

$$P = \frac{-E + TS + \mu N}{V}, \tag{44}$$

and take the partial derivative with respect to μ in order to check for thermodynamic consistency, it leads to

$$\left.\frac{\partial P}{\partial \mu}\right|_T = \frac{1}{V}\left[-\frac{\partial E}{\partial \mu} + T\frac{\partial S}{\partial \mu} + N + \mu\frac{\partial N}{\partial \mu}\right], = \frac{1}{V}\left[N + \sum_i\left(-\frac{T}{q-1}\left[1 + (q-1)\frac{E_i - \mu}{T}\right]\frac{\partial f_i^q}{\partial \mu} + \frac{Tq(1-f_i)^{q-1}}{q-1}\frac{\partial f_i}{\partial \mu}\right)\right], \tag{45}$$

then, by explicit calculation

$$\frac{\partial f_i^q}{\partial\mu} = \frac{q f_i^{q+1}}{T}\left[1+(q-1)\frac{E_i-\mu}{T}\right]^{-1+\frac{1}{1-q}},$$

$$\frac{\partial f_i}{\partial\mu} = \frac{f_i^2}{T}\left[1+(q-1)\frac{E_i-\mu}{T}\right]^{-1+\frac{1}{1-q}},$$

and

$$(1-f_i)^{q-1} = f_i^{q-1}\left[1+\frac{(q-1)(E_i-\mu)}{T}\right].$$

Introducing this into Eq. (45), yields

$$\left.\frac{\partial P}{\partial\mu}\right|_T = n, \qquad (46)$$

which proves the thermodynamic consistency (38).

We also calculate explicitly the relation in Eq. (36) can be rewritten as

$$\left.\frac{\partial E}{\partial S}\right|_n = \frac{\frac{\partial E}{\partial T}dT+\frac{\partial E}{\partial\mu}d\mu}{\frac{\partial S}{\partial T}dT+\frac{\partial S}{\partial\mu}d\mu} = \frac{\frac{\partial E}{\partial T}+\frac{\partial E}{\partial\mu}\frac{d\mu}{dT}}{\frac{\partial S}{\partial T}+\frac{\partial S}{\partial\mu}\frac{d\mu}{dT}}, \qquad (47)$$

since n is kept fixed one has the additional constraint

$$dn = \frac{\partial n}{\partial T}dT + \frac{\partial n}{\partial\mu}d\mu = 0,$$

leading to

$$\frac{d\mu}{dT} = -\frac{\frac{\partial n}{\partial T}}{\frac{\partial n}{\partial\mu}}. \qquad (48)$$

Now, we rewrite (47) and (48) in terms of the following expressions

$$\frac{\partial E}{\partial T} = \sum_i qE_if_i^{q-1}\frac{\partial f_i}{\partial T},$$

$$\frac{\partial E}{\partial\mu} = \sum_i qE_if_i^{q-1}\frac{\partial f_i}{\partial\mu},$$

$$\frac{\partial S}{\partial T} = \sum_i q\left[\frac{-f_i^{q-1}+(1-f_i)^{q-1}}{q-1}\right]\frac{\partial f_i}{\partial T},$$

$$\frac{\partial S}{\partial\mu} = \sum_i q\left[\frac{-f_i^{q-1}+(1-f_i)^{q-1}}{q-1}\right]\frac{\partial f_i}{\partial\mu},$$

$$\frac{\partial n}{\partial T} = \frac{1}{V}\left[\sum_i qf_i^{q-1}\frac{\partial f_i}{\partial T}\right],$$

and

$$\frac{\partial n}{\partial\mu} = \frac{1}{V}\left[\sum_i qf_i^{q-1}\frac{\partial f_i}{\partial\mu}\right].$$

By introducing the above relations into Eq. (47), the numerator of Eq. (47) becomes

$$\frac{\partial E}{\partial T}+\frac{\partial E}{\partial\mu}\frac{d\mu}{dT} = \sum_i qE_if_i^{q-1}\frac{\partial f_i}{\partial T} - \frac{\sum_{i,j} q^2E_i(f_if_j)^{q-1}\frac{\partial f_i}{\partial\mu}\frac{\partial f_j}{\partial T}}{\sum_j qf_j^{q-1}\frac{\partial f_j}{\partial\mu}} = \frac{\sum_{i,j} qE_i(f_if_j)^{q-1}C_{ij}}{\sum_j f_j^{q-1}\frac{\partial f_j}{\partial\mu}}. \qquad (49)$$

Where the abbreviation

$$C_{ij} \equiv (f_if_j)^{q-1}\left(\frac{\partial f_i}{\partial T}\frac{\partial f_j}{\partial\mu} - \frac{\partial f_j}{\partial T}\frac{\partial f_i}{\partial\mu}\right), \qquad (50)$$

has been introduced. One can rewrite the denominator part of Eq. (47) as

$$\frac{\partial S}{\partial T}+\frac{\partial S}{\partial\mu}\frac{d\mu}{dT} = \frac{\sum_{i,j}\frac{q}{(q-1)}\left[-f_i^{q-1}+(1-f_i)^{q-1}\right]f_j^{q-1}C_{i,j}}{\sum_j f_j^{q-1}\frac{\partial f_j}{\partial\mu}} = \frac{\sum_{i,j}\frac{q}{T}(E_i-\mu)(f_if_j)^{q-1}C_{i,j}}{\sum_j f_j^{q-1}\frac{\partial f_j}{\partial\mu}}, \qquad (51)$$

where

$$\frac{-f_i^{q-1}+(1-f_i)^{q-1}}{q-1} = \frac{(E_i-\mu)}{T}f_i^{q-1},$$

hence, by substituting Eqs. (49) and (51) in to Eq. (47), we find

$$\left.\frac{\partial E}{\partial S}\right|_n = T\frac{\sum_{i,j}E_iC_{ij}}{\sum_{i,j}(E_i-\mu)C_{ij}}, \qquad (52)$$

since $\sum_{i,j} C_{ij} = 0$, this finally leads to the desired result

$$\left.\frac{\partial E}{\partial S}\right|_n = T. \qquad (53)$$

Hence thermodynamic consistency is satisfied.

It has thus been shown that the definitions of temperature and pressure within the Tsallis formalism for non-extensive statistics lead to expressions which satisfy consistency with the first and second laws of thermodynamics. The remaining relations can be be shown to be satisfied in a similar manner.

3 Transverse Momentum Distributions: Fit Details

The total number of particles is given by the integral version of Eq. ((23)),

$$N = gV\int\frac{d^3p}{(2\pi)^3}\left[1+(q-1)\frac{E-\mu}{T}\right]^{-q/(q-1)} . \quad (54)$$

The corresponding (invariant) momentum distribution deduced from the equation above is given by

$$E\frac{d^3N}{d^3p} = gV E\frac{1}{(2\pi)^3}\left[1+(q-1)\frac{E-\mu}{T}\right]^{-q/(q-1)} . \quad (55)$$

In terms of the rapidity and transverse mass variables this becomes

$$\frac{d^2N}{dy\, p_T dp_T} = gV\frac{m_T\cosh y}{(2\pi)^2} \times \left[1+(q-1)\frac{m_T\cosh y-\mu}{T}\right]^{-q/(q-1)} \quad (.56)$$

At mid-rapidity $y = 0$ and for zero chemical potential this reduces to the following expression

$$\left.\frac{d^2N}{dp_T\, dy}\right|_{y=0} = gV\frac{p_T m_T}{(2\pi)^2}\left[1+(q-1)\frac{m_T}{T}\right]^{-q/(q-1)} . \quad (57)$$

In Fig. 4 we show a fit to the transverse momentum distributions obtained in p_p collisions at 900 GeV for identified particles π^-, K^-, $\bar{p}$ published by the ALICE collaboration [3]. We have also checked that the χ^2 values are of a similar quality. We have also made fits using the Tsallis-B distribution to experimental measurements published by the CMS collaboration [5]. These are shown in Figs. 5, 6 and 7 and are comparable with those shown by the CMS collaboration [5]. The resulting parameters are collected in Table 1. The most striking feature is that the values of the parameter q are fairly stable in the range 1.1 to 1.2 for all particles considered at 0.9 TeV. The temperature T cannot be determined very accurately for all hadrons but they are consistent with a value around 70 MeV.

For clarity we show these results also in Fig. 8 for the values of the parameter q and in Fig. 9 for the values of the Tsallis parameter T. The striking feature is that the values of q are consistently between 1.1 and 1.2 for all species of hadrons at 0.9. The values obtained for the temperature are clearly below values for the thermal freeze-out temperature that have been reported elsewhere in the literature. This is unavoidable when using the Tsallis distribution: for the same value of T, the Tsallis distribution is always higher than the Boltzmann distribution, hence, to reproduce the same transverse momentum, one has to use a lower temperature for the Tsallis distribution than for the Boltzmann one.

the ALICE [27], ATLAS [4] and CMS [26] collaborations have published data on the transverse momentum distribution of charged particles. These extend to much higher values of the transverse momentum [4] and would provide an important test for distinguishing formula (1) from (3). Since this does not involve identified particles, it makes use of a summation over several hadrons, e.g. pions, kaons and protons, hence the analysis is a bit more involved and will be considered in a separate publication. For completeness we also show the value of the volume V appearing in Eq. (1). The resulting radius R is shown in Fig. 10. If all hadrons originate from the same system and if there were no extra contributions coming from heavier resonances decaying into hadrons, then this volume should be the same for all hadrons. This is clearly

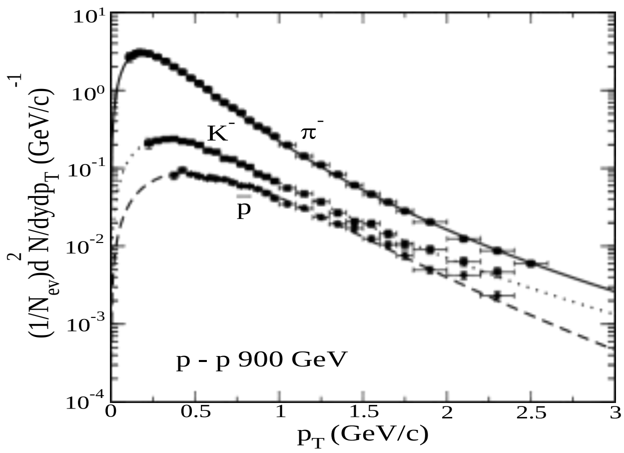

Fig. 4. Comparison between the measured transverse momentum distribution for π^-, K^- and $\bar{p}$ as measured by the ALICE collaboration [3] and the Tsallis-B distribution. The lines are fits using the parameterization given in Eq. (57) to the 0.9 TeV data with the parameters listed in Table 1. Solid line is for π^-, the dotted line is for K^-, the dashed line is for anti-protons.

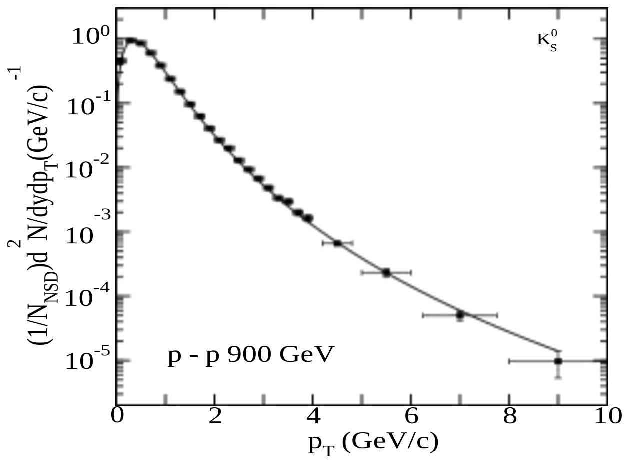

Fig. 5. Comparison between the measured transverse momentum distribution for K^0_S as measured by the CMS collaboration [5] and the Tsallis-B distribution. The solid line is a fit using the parameterization given in Eq. (57) to the 0.9 TeV data with the parameters listed in Table 1.

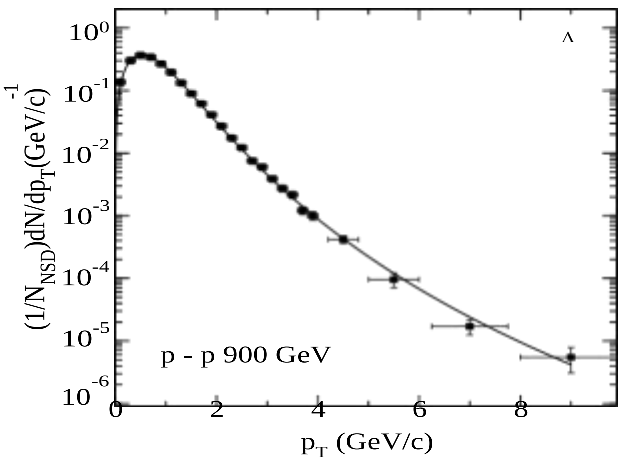

Fig. 6. Comparison between the measured transverse momentum distribution for Λ as measured by the CMS collaboration [5] and the Tsallis-B distribution. The solid line is a fit using the parameterization given in Eq. (57) to the 0.9 TeV data with the parameters listed in Table 1.

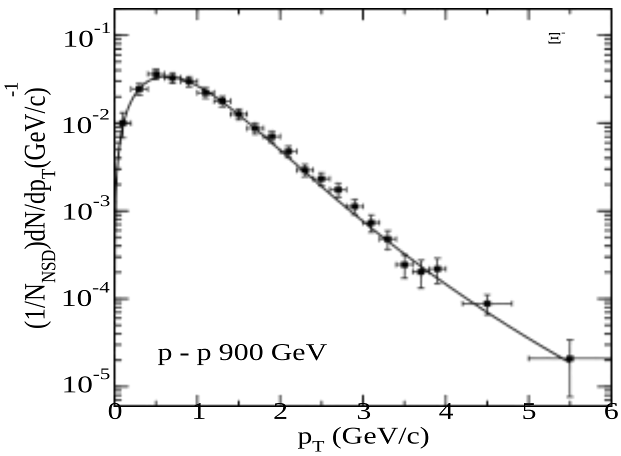

Fig. 7. Comparison between the measured transverse momentum distribution for Ξ^- as measured by the CMS collaboration [5] and the Tsallis-B distribution. The solid line is a fit using the parameterization given in Eq. (57) to the 0.9 TeV data with the parameters listed in Table 1.

not the case in the present analysis, in fact the radius is surprisingly larger which can only be interpreted by a very large time between chemical and thermal freeze-out. This clearly needs further investigation.

4 Discussion and Conclusions

In this paper we have presented a detailed derivation of the quantum form of the Tsallis distribution and proven the thermodynamic consistency of the resulting distribution. It was emphasized that an additional power of q is needed to achieve consistency with the laws of thermodynamics [13]. The resulting distribution was compared with recent measurements from the ALICE [3] and CMS

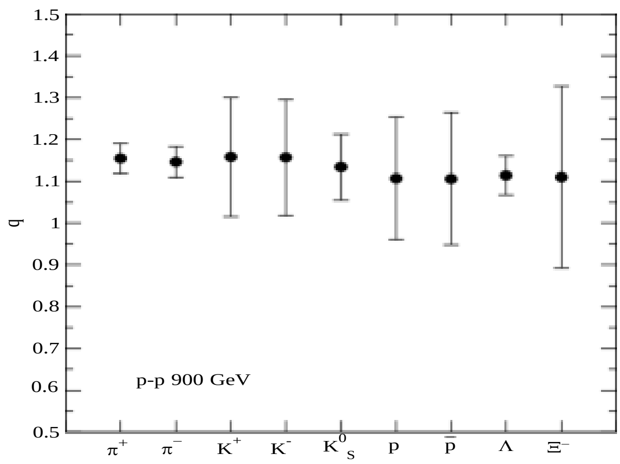

Fig. 8. Values of the Tsallis parameter q for different species of hadrons.

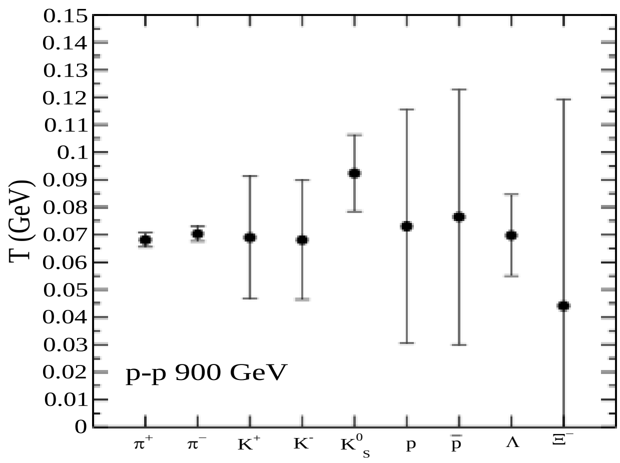

Fig. 9. Values of the Tsallis temperature T for different species of hadrons.

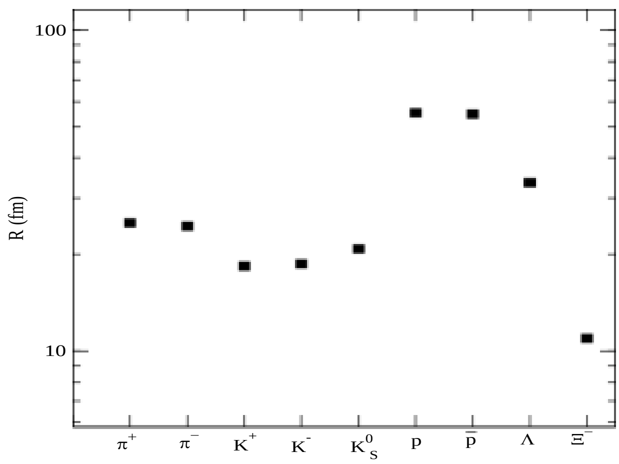

Fig. 10. Values of the radius R deduced from the volume V appearing in (1) for different particle species in $p-p$ collisions at 900 GeV.

Particle	q	T (GeV)	χ^2/ndf
π^+	1.154 ±0.036	0.0682 ±0.0026	12.01/30
π^-	1.146 ±0.036	0.0704 ± 0.0027	13.28/30
K^+	1.158 ±0.142	0.0690 ±0.0223	16.25/24
K^-	1.157 ±0.139	0.0681 ± 0.0217	7.06/24
K^0_S	1.134 ±0.079	0.0923 ±0.0139	14.41/21
p	1.107 ±0.147	0.0730 ± 0.0425	14.77/21
$\bar{p}$	1.106 ±0.158	0.0764 ±0.0464	13.18/21
Λ	1.114 ±0.047	0.0698 ± 0.0148	8.45/21
Ξ^-	1.110 ±0.218	0.0440 ± 0.0752	10.09/21

Table 1. Fitted values of the T and q parameters for different species of hadrons measured by the ALICE [3] and CMS collaborations [5], together with the corresponding χ^2 values, using the Tsallis-B form for the momentum distribution.

collaborations [5] and good agreement was obtained. The resulting parameter q which is a measure for the deviation from a standard Boltzmann distribution was found to be in the range 1.1-1.2. The resulting values of the temperature are also consistent within the errors and lead to a value of around 70 MeV. The analysis presented here cannot be considered complete as several elements are still missing. Most important is the contribution of heavier resonance which contribute to the final number of pions and kaons. Their effect on heavier baryons like the Ξ is not as large as for pions but nevertheless it is a factor that has to be taken into account. This could change the conclusions presented here.

References

1. B. I. Abelev, et al. (STAR Collaboration), Phys. Rev. C75, 064901 (2007)
2. A. Adare et al. (PHENIX Collaboration), Phys. Rev. C83, 064903 (2011)
3. K. Aamodt, et al. (ALICE Collaboration), Eur. Phys. J. C71 1655 (2011)
4. G. Aad, et al. (ATLAS Collaboration), New J. Phys. 13 (2011) 053033.
5. V. Khachatryan, et al. (CMS Collaboration), JHEP 05, 064 (2011)
6. C. Tsallis, J.Statist.Phys. 52, 479 (1988)
7. C. Tsallis, R. S. Mendes, A. R. Plastino, Physica A261, 534 (1998)
8. T.S. Biró, "Is There a Temperature?", Springer Verlag (2011).
9. T.S. Biró, G. Purcel, K. Ürmössy, Eur. Phys. J. A40 (2009) 325.
10. G. Wilk and Z. Wlodarczyk, Eur. Phys. J. A40 (2009) 299; Phys. Rev. C79 (2009) 54903..
11. F. Pereira, R. Silva, J. Alcaniz, Phys. Rev. C76, 015201 (2007)
12. F. Pereira, R. Silva, J. Alcaniz, Phys. Lett. A373, 4214 (2009), 0906.2422
13. J. M. Conroy, H. G. Miller, A. R. Plastino, Phys. Lett. A374, 4581 (2010)
14. J. Cleymans and D. Worku, J. Phys. G39 (2012) 025006.
15. F. Buyukkilic, D. Demirhan, Phys.Lett. A181, 24 (1993)
16. F. Pennini, A. Plastino, A. R. Plastino, Phys. Lett. A208, 4, 309 (1995)
17. A. M. Teweldeberhan, A. R. Plastino, H. G. Miller, Phys.Lett. A343, 71 (2004)
18. J. M. Conroy, H. Miller, Phys. Rev. D78, 054010 (2008)
19. J. Chen, Z. Zhang, G. Su, et al., Phys. Lett. A 300, 1, 65 (2002)
20. J. Cleymans, G. Hamar, P. Lévai, S. Wheaton, J.Phys. G36, 064018 (2009)
21. T.S. Biró and E. Molnár, Phys. Rev. C 85, 024905 (2011).
22. T.S. Biró and P. Ván, Phys. Rev. E 83, 1061147 (2011)
23. T.S. Biró, G. Purcsel, Phys. Rev. Lett. 95, 162302 (2005)
24. G.G. Barnaföldi, K. Ürmössy, T.S. Biró, J. Phys. Conf. Ser. 270 (2011) 012008
25. S. R. de Groot, W. A. van Leeuwen, C. G. van Weert, Relativistic Kinetic Theory (North Holland, 1980)
26. CMS Collaboration, JHEP 08, 086 (2011).
27. K. Aamodt, ALICE Collaboration, Phys. Lett. B 693, 53 (2010).