NUMERICAL SOLUTION OF FLUID AND PARTICLE PHASE ON VELOCITY OF HEAT WITH EFFECT OF MAGNETIC FIELD IN INCOMPRESSIBLE DUSTY FLUID

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Abstract
In this chapter the steady fluid and particle phase of a fluid with SPM past a horizontal flat plate has been considered in a magnetic field. It is assumed that the free stream velocity of the fluid and particle far away from the plate is assumed to be U and particle concentration be $\rho_{p\infty}$. The cloud phase of particle is called pseudo fluid, which enables us to reduce its governing equations similar to Navier Stokes equations. The result obtaining by applying method of the finite difference and the magnitude of the particle reduced significantly. Although some of our numerical solutions agree with some of the available results in the literature review, other results differs because of the effect of the introduced magnetic field
Keywords: Induced Magnetic field, Differential equations, Dusty fluid flow, incompressible fluid,

Introduction
In this chapter the steady laminar flow of a fluid with SPM past a horizontal flat plate has been considered in a magnetic field. It is assumed that the free stream velocity of the fluid and particle far away from the plate is assumed to be U and particle concentration be $\rho_{p\infty}$. In the context of the continuum hypothesis, the solid particle phase is seen as a cloud of particles, each of which is perfectly spherical and has the same radius. We may simplify the governing equations of the cloud phase of particles, which we refer to as pseudo fluid, to the form of Navier Stokes equation. Here this case of a dilute suspension, the stokes law may be obtained from the Boltzmann equations that regulate the particle phase. The particles will not disturb the basic fluid motion but diffuse through the carrier fluid. Here we consider gravitational force as a body force and the buoyancy as a part of the interaction force between the pseudo fluid and fluid phase. For pseudo fluid the total force due to gravitational acceleration is given by $\phi(\rho_{sp} - \rho)$ g and for the gas total force due to gravitational acceleration by ρg.

MATHEMATICAL FORMULATIONS
After introducing the non-dimensional variables, the governing equations are

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \quad (1)$$

$$u\frac{\partial \rho_p}{\partial x}+v\frac{\partial \rho_p}{\partial y}=\frac{\varepsilon}{R_e}\frac{\partial^2 \rho_p}{\partial y^2} \quad (2)$$

$$(1-\phi)\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{1}{R_e}\frac{\partial^2 u}{\partial y^2}+f\,\alpha\rho_p\left(u_p-u\right)-\frac{G_r\theta}{R_e^2} \quad (3)$$

$$\left(u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y}\right)=\frac{-\varepsilon}{R_e}\frac{\partial^2 u_p}{\partial y^2}+f\left(u-u_p\right)+\frac{1}{F_r}-\frac{G_r\theta}{\alpha\rho_{sp}R_e^2} \quad (4)$$

$$\left(u_p\frac{\partial v_p}{\partial x}+v_p\frac{\partial v_p}{\partial y}\right)=\frac{-\varepsilon}{R_e}\frac{\partial^2 v_p}{\partial y^2}+f\left(v-v_p\right) \quad (5)$$

$$(1-\phi)\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right)=\frac{1}{P_r R_e}\left(\frac{\partial^2\theta}{\partial y^2}\right)+\frac{2\alpha\rho_p f}{3P_r}\left(\theta_p-\theta\right) \qquad (6)$$

$$\phi\left(u_p\frac{\partial\theta_p}{\partial x}+v_p\frac{\partial\theta_p}{\partial y}\right)=\frac{2f}{3P_r}\left(\theta-\theta_p\right) \qquad (7)$$

These are the boundary condition

$u = 0,\ v = 0,\ \rho_p = A(x),\ u_p = B(x),\ \theta = 1,\ \theta_p = 1$ when $y=0$

$u = 1,\ v_p = 0,\ \rho_p = 1,\ u_p = 1,\ \theta = 0,\ \theta_p = 0$ when $y=1$

SOLUTION

By applying the Crank-Nicholson finite difference technique, we may transform the fluid phase's continuity equation (1) into its equivalent difference equation. The different steps are given below:

$$\left(\frac{\partial u}{\partial x}\right)_{m+\frac{1}{2},n}+\left(\frac{\partial v}{\partial y}\right)_{m+\frac{1}{2},n}=0$$

$$\Rightarrow v_{m+\frac{1}{2},n}=v_{m+\frac{1}{2},n-1}-\frac{\Delta y}{2\Delta x}A \qquad (8)$$

Where $A=\left(u_{m+1,n-1}-u_{m,n-1}+u_{m+1,n}-u_{m,n}\right)$

It is solved subject to the boundary condition

$y = 0 \quad : v = 0$

The diffusion equation (2) for particle phase is replaced by

$$u_{m+\frac{1}{2},n}\left(\frac{\partial\rho_p}{\partial x}\right)_{p+\frac{1}{2},q}+v_{p+\frac{1}{2},q}\left(\frac{\partial\rho_p}{\partial y}\right)_{p+\frac{1}{2},q}=\frac{\varepsilon}{R_e}\left(\frac{\partial^2\rho_p}{\partial y^2}\right)_{p+\frac{1}{2},q}$$

and using the technique of Crank-Nicholson it can be expressed as follows:

$$\frac{1}{2}\left(u_{m+1,n}+u_{m,n}\right)\cdot\frac{1}{\Delta x}\left(\rho_{p_{m+1,n}}-\rho_{p_{m,n}}\right)$$
$$+v_{m+\frac{1}{2},n}\left\{\frac{1}{4\Delta y}\left(\rho_{p_{m+1,n+1}}-\rho_{p_{m+1,n-1}}+\rho_{p_{m,n+1}}-\rho_{p_{m,n-1}}\right)\right\}$$
$$=\frac{\varepsilon}{2R_e(\Delta y)^2}\left(\rho_{p_{m+1,n+1}}-2\rho_{p_{m+1,n}}+\rho_{p_{m+1,n-1}}+\rho_{p_{m,n+1}}-2\rho_{p_{m,n}}+\rho_{p_{m,n-1}}\right)$$

which is simplified to the form:

$$A_n\rho_{p_{m+1,n-1}}+B_n\rho_{p_{m+1,n}}+C_n\rho_{p_{m+1,n+1}}=D_n \qquad (9)$$

where

$$A_n=-\frac{v_{m+\frac{1}{2},n}}{4\Delta y}-\frac{\varepsilon}{2R_e(\Delta y)^2}$$

$$B_n=\frac{k}{2\Delta x}+\frac{\varepsilon}{R_e(\Delta y)^2},\ \text{where}\ k=\left(u_{p+1,q}+u_{p,n}\right)$$

$$C_n=\frac{v_{m+\frac{1}{2},n}}{4\Delta y}-\frac{\varepsilon}{2R_e(\Delta y)^2}$$

$$D_{na=} = \frac{A}{2\Delta x} - \frac{B}{4\Delta y}$$
$$+\frac{\varepsilon}{2R_e(\Delta y)^2}\left(\rho_{p_{m,n+1}} - 2\rho_{p_{m,n}} + \rho_{p_{m,n-1}}\right)$$
$$A = \left(u_{m+1,n} + u_{m,n}\right)\rho_{p_{m,n}} \; and$$
$$b = \left(\rho_{p_{m,n+1}} - \rho_{p_{m,n-1}}\right)v_{m+\frac{1}{2},n}$$

Using the formula presented by Jain and Ghosh (1979). The initial velocity profile for ρ_p is found by

$$\rho_p = 1-(1-y)^3(1-A(x))$$

where the boundary conditions

$$:\rho_p = A(x) : y = 0$$
$$\rho_p = 1, \qquad y=1$$

are satisfied.

The momentum equation (3) for fluid phase is reduced to its finite difference form as follows:

$$(1-\phi)\left[u_{m+\frac{1}{2},n}\left(\frac{\partial u}{\partial x}\right)_{m+\frac{1}{2},n} + v_{m+\frac{1}{2},n}\left(\frac{\partial u}{\partial y}\right)_{m+\frac{1}{2},n}\right]$$
$$= \frac{1}{R_e}\left(\frac{\partial^2 u}{\partial y^2}\right)_{m+\frac{1}{2},n} + \alpha f\,\rho_{p_{p+\frac{1}{2},q}}\left(u_{p_{m+\frac{1}{2},q}} - u_{p+\frac{1}{2},q}\right) - \frac{G_r}{R_e^2}\theta p+\tfrac{1}{2},q$$
$$\Rightarrow (1-\phi)\left[\frac{1}{2}\left(u_{m+1,n}+u_{m,n}\right)\cdot\frac{1}{\Delta x}\left(u_{m+1,n}-u_{m,n}\right)\right.$$
$$\left.+v_{m+\frac{1}{2},n}\left\{\frac{1}{4\Delta y}m\right\}\right] where\, m = \left(u_{m+1,n+1} - u_{m+1,n-1} + u_{m,n+1} - u_{m,n-1}\right)$$
$$= \frac{1}{2R_e(\Delta y)^2}\left(u_{m+1,n+1} - 2u_{m+1,n} + u_{m+1,n-1} + u_{m,n+1} - 2u_{m,n} + u_{m,n-1}\right)$$
$$+\frac{f\alpha\left(\rho_{p_{m+1,n}} + \rho_{p_{m,n}}\right)\left(u_{p_{m+1,n}} + u_{p_{m,n}} - u_{m+1,n} - u_{m,n}\right)}{4} - \frac{G_r}{R_e^2}\theta m+\tfrac{1}{2},n$$

This is simplified to the following form

$$A_n^* u_{m+1,n-1} + B_n^* u_{m+1,n} + C_n^* u_{m+1,n+1} = D_n^* \qquad (10)$$

Where, $A_n^* = -\dfrac{(1-\phi)v_{m+\frac{1}{2},n}}{4\Delta y} - \dfrac{1}{2R_e(\Delta y)^2}$

$$B_n^* = \frac{f}{2\Delta x} + \frac{1}{R_e(\Delta y)^2} + \frac{\alpha f\left(\rho_{p_{m+1,n}} + \rho_{p_{m,n}}\right)}{4},\; f = (1-\phi)u_{m+1,n}$$
$$C_n^* = \frac{(1-\phi)v_{m+\frac{1}{2},n}}{4\Delta y} - \frac{1}{2R_e(\Delta y)^2}$$

$$D_n^* = \frac{(1-\phi)u_{m,n}^2}{2\Delta x} - \frac{x}{4\Delta y}$$

$$+\frac{1}{2R_e(\Delta y)^2}\left(u_{m,n+1} - 2u_{m,n} + u_{m,n-1}\right)$$

$$+\frac{\alpha f\left(\rho_{p_{m+1,n}} + \rho_{p_{m,n}}\right)\left(u_{p_{m+1,n}} + u_{p_{m,n}} - u_{p,q}\right)}{4} - \frac{G_r}{R_e^2}\theta p + \tfrac{1}{2}, q$$

$$x = (1-\phi)\left(u_{m,n+1} - u_{m,n-1}\right)v_{m+\frac{1}{2},n}$$

The answer is found in the boundary conditions:

u = 0 when y=0

u = 1 when y=1

To determine the initial velocity profile for u, Jain and Ghosh (1979) propose the following:

$$u = 1-(1-y)^4$$

As an alternative to the x-axis version of the momentum equation for particles (see (4)), we have

$$\left[u_{p_{m+\frac{1}{2},n}}\left(\frac{\partial u_p}{\partial x}\right)_{m+\frac{1}{2},n} + v_{p_{m+\frac{1}{2},n}}\left(\frac{\partial u_p}{\partial y}\right)_{m+\frac{1}{2},n}\right]$$

$$= \frac{-\varepsilon}{R_e}\left(\frac{\partial^2 u_p}{\partial y^2}\right)_{m+\frac{1}{2},n} + f\left(u_{m+\frac{1}{2},n} - u_{p_{m+\frac{1}{2},n}}\right) + \frac{1}{F_r} - \frac{G_r}{\alpha\rho_{sp}R_e^2}\theta m + \tfrac{1}{2}, n$$

$$\Rightarrow\left[\frac{1}{2}A + v_{m+\frac{1}{2},n}\left\{\frac{1}{4\Delta y}B\right\}\right]$$

$$A = \left(u_{p_{m+1,n}} + u_{p_{m,n}}\right)\cdot\frac{1}{\Delta x}\left(u_{p_{m+1,n}} - u_{p_{m,n}}\right)$$

$$B = \left(u_{p_{m+1,n+1}} - u_{p_{m+1,n-1}} + u_{p_{m,n+1}} - u_{p_{m,n-1}}\right)$$

This is simplified to the form:

$$A_n^{**}u_{p_{m+1,n-1}} + B_n^{**}u_{p_{m+1,n}} + C_n^{**}u_{p_{m+1,n+1}} = D_n^{**} \qquad (11)$$

Where,

$$A_n^{**} = -\frac{v_{m+\frac{1}{2},n}}{4\Delta y} + \frac{\varepsilon}{2R_e(\Delta y)^2}$$

$$B_n^{**} = \frac{u_{p_{m+1,n}}}{2\Delta x} - \frac{\varepsilon}{R_e(\Delta y)^2} + \frac{f}{2}$$

$$C_n^{**} = \frac{v_{m+\frac{1}{2},n}}{4\Delta y} + \frac{\varepsilon}{2R_e(\Delta y)^2}$$

$$D_n^{**} = \frac{u_{p_{m,n}}^2}{2\Delta x} - \frac{\left(u_{p_{m,n+1}} - u_{p_{m,n-1}}\right)v_{m+\frac{1}{2},n}}{4\Delta y}$$

$$-\frac{\varepsilon}{2R_e(\Delta y)^2}\left(u_{p_{m,n+1}} - 2u_{p_{m,n}} + u_{p_{m,n-1}}\right) + \frac{f}{2}\left(u_{m+1,n} + u_{m,n} - u_{p_{m,n}}\right)$$

$$+\frac{1}{F_r} - \frac{G_r}{\alpha\rho_{sp}R_e^2}\theta m + \tfrac{1}{2}, n$$

Since $v_p \approx v$, so $v_{p_{m+\frac{1}{2},n}}$ is replaced by $v_{m+\frac{1}{2},n}$

$y = 0$; $u_p = B(x)$
$y = 1$; $u_p = 1$

The equation of momentum (5) for the particle phase in the y-direction is replaced by:

$$\left[u_{p_{m+\frac{1}{2},n}} \left(\frac{\partial v_p}{\partial x} \right)_{m+\frac{1}{2},n} + v_{p_{m+\frac{1}{2},n}} \left(\frac{\partial v_p}{\partial y} \right)_{m+\frac{1}{2},n} \right] = \frac{-\varepsilon}{R_e} \left(\frac{\partial^2 v_p}{\partial y^2} \right)_{m+\frac{1}{2},n} + f\left(v_{m+\frac{1}{2},n} - v_{p_{m+\frac{1}{2},n}} \right)$$

The term $v_{p_{m+\frac{1}{2},n}}$ appeared in the second term is replaced by $v_{m+\frac{1}{2},n}$ to make the equation quasi-linear as $v_p \approx v$. Then using the technique of Crank-Nicholson it can be expressed as

$$\left[\frac{1}{2}\left(u_{p_{m+1,n}} + u_{p_{m,n}} \right) \frac{2\left(v_{p_{m+\frac{1}{2},n}} - v_{p_{m,n}} \right)}{\Delta x} + v_{m+\frac{1}{2},n} \left(\frac{v_{p_{m+\frac{1}{2},n+1}} - v_{p_{m+\frac{1}{2},n-1}}}{2\Delta y} \right) \right]$$
$$= \frac{-\varepsilon}{R_e} \left(\frac{v_{p_{m+\frac{1}{2},n+1}} - 2v_{p_{m+\frac{1}{2},n}} + v_{p_{m+\frac{1}{2},n-1}}}{(\Delta y)^2} \right) + f\left(v_{m+\frac{1}{2},n} - v_{p_{m+\frac{1}{2},n}} \right)$$

After replacing $v_{p_{m,n}}$ by $v_{m+\frac{1}{2},n}$ as $v_p \approx v$ the above difference equation is simplified to the following form:

$$E_n v_{p_{m+\frac{1}{2},n-1}} + F_n v_{p_{m+\frac{1}{2},n}} + G_n v_{p_{m+\frac{1}{2},n+1}} = H_n \qquad (12)$$

Where,

$$E_n = -\frac{v_{m+\frac{1}{2},n}}{2\Delta y} + \frac{\varepsilon}{R_e (\Delta y)^2}$$

$$F_n = \frac{\left(u_{p_{m+1,n}} + u_{p_{m,n}} \right)}{\Delta x} - \frac{2\varepsilon}{R_e (\Delta y)^2} + f$$

$$G_n = \frac{v_{m+\frac{1}{2},n}}{2\Delta y} + \frac{\varepsilon}{R_e (\Delta y)^2}$$

Particle phase continuity equation is used to determine the compatibility criterion for Vp.

$$\frac{\partial}{\partial x}\left(\rho_p u_p \right) + \frac{\partial}{\partial y}\left(\rho_p v_p \right) = 0$$

When evaluated on the plate y = 0 we obtain;

$$\frac{\partial}{\partial x}\left[A(x) \cdot B(x) \right] + A(x) \frac{\partial v_p}{\partial y}\bigg|_{y=0} = 0$$

Which gives

$$v_{p_{m+\frac{1}{2},1}} - v_{p_{m+\frac{1}{2},-1}} = -\frac{2\Delta y}{A(x)} \cdot \frac{\partial}{\partial x}\left[A(x) \cdot B(x) \right]$$

$$\Rightarrow v_{p_{m+\frac{1}{2},-1}} = v_{p_{m+\frac{1}{2},1}} + R \qquad (13)$$

For n = 0 the equation (5) after the substitution reduces to

$$v_{p_{m+\frac{1}{2},0}} = -\left[\frac{E_0 R}{F_0} + \frac{(E_0 + G_0)}{F_0} v_{p_{m+\frac{1}{2},1}}\right]$$

So the boundary conditions for v_p are given by

$$y=0 \quad : \quad v_p = -\left[\frac{E_0 R}{F_0} + \frac{(E_0 + G_0)}{F_0} v_{p_{m+\frac{1}{2},1}}\right]$$

$$y=1 \quad : \quad v_p = 0$$

The energy equation (6) for fluid phase is replaced as:

$$(1-\phi)\left[u_{p+\frac{1}{2},q}\left(\frac{\partial\theta}{\partial x}\right)_{m+\frac{1}{2},n} + v_{p+\frac{1}{2},q}\left(\frac{\partial\theta}{\partial y}\right)_{p+\frac{1}{2},q}\right]$$

$$= \frac{1}{P_r R_e}\left(\frac{\partial^2\theta}{\partial y^2}\right)_{m+\frac{1}{2},n} + \frac{2\alpha f}{3P_r}\rho_{p_{m+\frac{1}{2},n}}\left(\theta_{p_{m+\frac{1}{2},n}} - \theta_{m+\frac{1}{2},n}\right)$$

The above difference equation can be expressed as:

$$E_n^*\theta_{m+1,n-1} + F_n^*\theta_{m+1,n} + G_n^*\theta_{m+1,n+1} = H_n^* \qquad (14)$$

Where,

$$E_n^* = -\frac{(1-\phi)v_{m+\frac{1}{2},n}}{4\Delta y} - \frac{1}{2P_r R_e (\Delta y)^2}$$

$$F_n^* = \frac{(1-\phi)(u_{m+1,n} + u_{m,n})}{2\Delta x} + \frac{1}{P_r R_e (\Delta y)^2} + \frac{\alpha f\left(\rho_{p_{m+1,n}} + \rho_{p_{m,n}}\right)}{6P_r}$$

$$G_n^* = \frac{(1-\phi)v_{m+\frac{1}{2},n}}{4\Delta y} - \frac{1}{2P_r R_e (\Delta y)^2}$$

Heat Transfer

In accordance with Newton's rule of cooling, at a given location x the amount of heat is transferred from the plate to fluid in terms of both area and time is equal to q(x).

$$q(x) = \alpha(x)(T_w - T_\infty),$$

Where $\alpha(x)$ is the coefficient of heat transfer and (T_w - T_∞) is the difference between the temperature of the plate and that of the fluid.

Since all heat transfer between the fluid and the plate occurs by conduction at the plate, we know from Fourier's law that:

$$q(x) = -K\left.\frac{\partial T}{\partial y}\right|_{y=0} .$$

Hence dimensionless coefficient of heat transfer, known as Nusselt number is given by

$$N_u = \frac{\alpha(x)L}{K} = \frac{q(x)L}{K(T_w - T_\infty)} = -\left.\frac{\partial\theta}{\partial y}\right|_{y=0} \quad \text{(after dropping bar)}$$

$$= \frac{(\theta_{0,n} - \theta_{1,n})}{\Delta y}$$

Skin Friction

$$\tau_w = \mu\left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{\mu U}{L}\left.\frac{\partial \bar{u}}{\partial \bar{y}}\right|_{\bar{y}=0}$$

$$C_{f_0} = \frac{2\tau_w}{\rho U^2} = \frac{2\mu U}{\rho U^2 L} \cdot \left.\frac{\partial \bar{u}}{\partial \bar{y}}\right|_{\bar{y}=0}$$

$$= \frac{2\upsilon}{UL} \cdot \left.\frac{\partial \bar{u}}{\partial \bar{y}}\right|_{\bar{y}=0} = \frac{2}{R_e} \cdot \left.\frac{\partial u}{\partial y}\right|_{y=0} \quad \text{(After dropping bar)}$$

$$= \frac{2}{R_e}\left(\frac{u_{1,n} - u_{0,n}}{\Delta y}\right) \qquad (9)$$

Table 1: The value of skin friction coefficient & Nusselt number at different values of X.

X	N_u	C_f
0.03	3273.02	0.65463
0.08	1196.96	0.39879
0.13	1572.18	0.31443
0.18	1340.01	0.26823
0.23	1182.61	0.23645
0.28	1069.49	0.21371
0.33	986.34	0.19735

Table 2: Differences in skin friction and the Nusselt number with varying Alpha

X	α	N_u	C_f
0.5	0.1	801.75	0.16038
0.5	0.2	801.73	0.16039
0.5	0.3	801.68	0.16040
0.5	0.4	801.69	0.16041
0.5	0.5	801.68	0.16041
0.5	0.6	801.64	0.16042

Table 3: Different values of ϕ and their effects on the Nusselt number and skin friction

X	ϕ	N_u	C_f
0.5	0.1	801.75	0.16038
0.5	0.2	801.85	0.16039
0.5	0.3	801.91	0.16039
0.5	0.4	801.85	0.16040
0.5	0.5	801.82	0.16040
0.5	0.6	801.83	0.16040

Table 4: Nusselt number and the skin friction Changes as the Reynolds number changes.

X	R_e	N_u	C_f
0.5	1000	801.89	1.60425
0.5	1500	801.72	1.06966
0.5	2000	801.13	0.80237
0.5	2500	801.63	0.64190
0.5	3000	801.73	0.53494
0.5	3500	801.22	0.45850

Table 5: Nusselt number and skin friction values for a range of P_r

X	P_r	N_u	C_f
0.5	0.72	801.75	0.16038
0.5	0.85	801.78	0.16038
0.5	1.00	801.62	0.16038
0.5	1.25	801.70	0.16039
0.5	1.50	801.53	0.16039
0.5	1.75	801.48	0.16039

CONCLUSION

Based on the data in table 1, we know that the Nusselt number and the skin friction coefficient are both positive and decrease with distance downstream. Based on the data in table 2, it can be shown that as increasesα, the Nusselt number continues to fall. However, α as becomes larger, the skin's friction coefficient also grows. According to the data in table 3, ϕ as rises, so does the Nusselt number and the skin friction coefficient. According to the data in table 4, when R_e rises, the Nusselt number and the skin friction coefficient fall. The skin friction coefficient remains constant while the Nusselt number drops as P_r rises (Table 5). The Nusselt number has been shown to rely insignificantly on the Grashoff number, but to depend significantly on the Reynolds number, which is always positive and decreases as one moves downstream. It is also deduced that fluids always transfer heat to plates

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