# AN INTEGRATED SINGLE-VENDOR, SINGLE- BUYER INVENTORY MODELFOR IMPERFECT QUALITY PRODUCTION, IMPERFECT INSPECTION AT VENDOR SITE

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## Introduction

Product quality was not faultless in conventional production/inventory models, according to (Salameh & Jaber, 2000). By accounting for defective items received from a specific vendor's probability density function, they improved the basic EOQ model. Before selling a large number of products, the seller performed a complete examination at a rate of x units per unit time, where x > D. (the demand rate). Items that were found to be faulty were held in inventory until the conclusion of each lot cycle and then sold at a discount to the general public. Assuming that buyers are prepared to wait for the next supply to arrive, (Wee et al., 2007) enlarged the EOQ model and allowed for backordering when there was a scarcity of items. An error in (Salameh & Jaber, 2000)'s EOQ model was addressed utilising the renewal andreward theorem by (Maddah & Jaber, 2008). As a result, the company was able to calculate its expected profit with precision. According to Khan et al. (2011), the inspection process was flawed. It's possible that items are incorrectly grouped. It's possible to classify non-faulty objects as non-defective and defective things as non- defective. Consumers returned faulty products sold in the market for replacement withfresh items that were retained in the inventory for future purchases by customers due to an inspection mistake. At the completion of each inspection operation, all defectiveproducts returned by consumers and things found to be defective were returned to the vendor for destruction. Inspection errors increased inventory levels and increased demand for products, while classifying non-defective items as defective resulted in lost profits due to decreased prices being charged for non-defective items. Hsu and Hsu (2012) corrected an assumption by Wee and colleagues (2007) that all backorders were cleared immediately after a new batch of products arrived and ignored the time necessary for item inspection.

The preceding works assumed that the things were inspected at the buyer's location, as originally recommended by (Salameh & Jaber, 2000). Other subsequent study papers (Salameh & Jaber, 2000) used the same premise. In this study endeavour, the assumption has been revised to assume that the vendor will check things in addition toproducing them.

## The Mathematical Model's Notation and Assumptions

QP : the number of units in a lot generated each manufacturing cycle
Q : the number of products in a lot provided by the seller to the customer
n : a positive integer representing the number of deliveries to the buyer everymanufacturing cycle$(QP = nQ)$
D : the annual pace of item demand
P : the pace at which goods are produced (P > D)x : the rate at which objects are inspected x > P Sv : the cost of setup each manufacturing cycle
K : the buyer's ordering cost per order
Ci : the inspection fee charged by the seller per unit
Cw : the vendor's per-unit cost of manufacturing faulty products (warranty cost)
$C\alpha\beta$ :the buyer's cost per unit in the market for selling damaged products (due to TypeII Error)
Cav : the vendor's market cost of selling damaged products (due to Type II Error)

Cr : the expense incurred by the vendor in rejecting non-defective items as defectiveones
hv : the vendor's inventory holding cost per unit itemhb : the buyer's inventory holding cost per unit item F : the cost of transportation for each item deliveredp : the likelihood of producing damaged products
*f* (p) : p's probability density function
e1: the probability of type I inspection Error (Classifying Non-Defective item asDefective items)
*f* (e1) : e1's probability density function
e2: the likelihood of type II inspection error (Classifying Defective item as Non- Defective items)
*f* (e2) : e2's probability density function
B1 : the amount of faulty goods identified following inspection per manufacturingbatch
B2 : the amount of faulty goods categorised as Non-Defective items in a singlemanufacturing lot (Type II Inspection Error)
T : the time elapsed between two subsequent delivery of Q products to purchasersT1 : the time frame in which the seller manufactures things
T2 : the time span in which the merchant distributes products from inventoryTC : the duration of the production cycle (TC = nT)
* : the superscript to reflect the best possible value

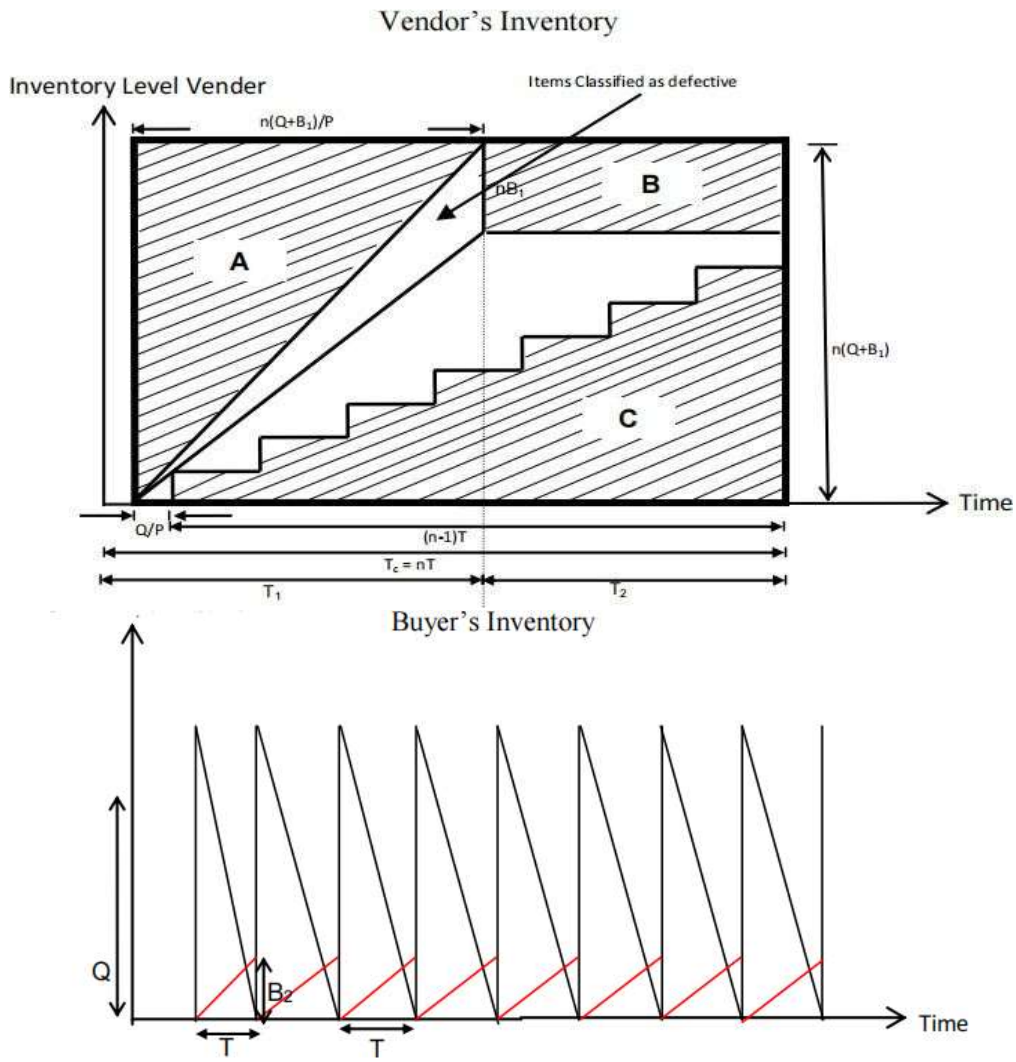
Single-vendor and single buyer paradigm assumes that items are made by the seller. Pis bigger than D in industrial industries because of the higher pace of output. A single production setup cycle is used to produce many batches of items, which are then delivered to the customer following inspection. Manufacturing industries' production processes involve machine and human faults, are of poor quality, and can produce some defective items with a specified probability distribution and a probability p f(p).Before distributing things in lots to the customer, all items are inspected to remove B1 faulty items. To save time, the inspection is performed concurrently with manufacturing. x is greater than the pace of item manufacture P (x > P), ensuring that goods produced are examined without delay in manufacturing businesses. Inspectionsare also subject to error due to the presence of human factors, and as a result, they may uncover any of the following two categories of defects:

Type I : With probability e1 and probability density function f, the inspector wronglyidentifies non-faulty things as defective (e1)
Type II : With probability e2 and probability density function f, the inspector wronglylabels faulty objects as non-defective items (e2)

When an inspector makes a type I error, he or she deems a non-faulty item to be defective. It results in a revenue loss of Cj per unit item since some non-defective (labelled as faulty) when disposed of in bulk alongside other damaged goods, theyreceive a reduced price.

During a type II inspection error, an inspector incorrectly classifies an item as non- defective. These B2 products are provided to the buyer in each lot to be sold at the market. Customers identify flaws in such things while using them. Clients come to the buyer to get a warranty replacement for a broken goods. B2 Each lot cycle ends with a return to the vendor of any defective items that were purchased from customers. The
vendor subsequently disposed of these B2 products at a discounted rate shortly afterreceiving them from the buyer. The cost to the consumer is $C\alpha\beta$ and the cost to the vendor is Cav for selling a faulty item per unit.

**Analytical Framework**




$Q + B_1$ is the number of faulty products generated in a production lot with theprobability distribution p

$= (Q + B_1)p$ (4.1)

The number of non-defective items produced in a manufacturing lot $Q + B_1$ given theprobability distribution p of producing defective items is

$= (Q + B_1)(1-p)$ (4.2)

The number of items categorised as defective that are really non-faulty as a result of Type I inspection Error $e_1$ in a production lot $Q + B_1$, and the probability distributionp to create defective items in production is

(Non-Defective □ Defective).

$= (Q + B_1)(1-p)e_1$ (4.3)

The number of items identified as non-faulty that are really defective owing to Type IIinspection Error $e_2$ in a manufacturing lot $Q + B_1$ is

(Defective □ Non-Defective).

$= (Q + B_1)pe_2$ (4.4)

Considering Type I inspection Error $e_1$ and Type II inspection Error $e_2$ in a productionlot $Q + B_1$, the probability distribution p to create faulty products in production is (Defective □ Defective).

$$= (Q + B_1)p(1-e_2) \tag{4.5}$$

Considering Type I inspection Error $e_1$ and Type II inspection Error $e_2$ in a production lot $Q + B_1$, the probability distribution p to create non-faulty products in production is (Non-Defective □ Non-Defective).

$$= (Q + B_1)\{(1-p)(1-e_2)\} \tag{4.6}$$

$B_1$ is the number of objects identified as faulty as a result of a type I inspection mistake. The vendor produces $B_1$ things in addition to Q products in order to offer Q goods to the customer at the start of each replenishment cycle. As a result, the vendor produced $(Q + B_1)$ things for each delivery of Q items.

Thus $B_1$= Defective items classified as defective + Type I error (classify non-defective as defective)

$$= (Q + B_1)p(1-e_2) + (Q + B_1)(1-p)e_1 \tag{4.7}$$

following its resolution,

$$B_1 = \frac{(1-e_2) + (1-P)e_1}{1 - \{(1-e_2\ \ ) + (1-P))_1\}}\, Q \tag{4.8}$$

$B_2$ goods are faulty owing to a type II error. (Mark faulty as non defective)

$$= (Q + B_1)pe_2 \tag{4.9}$$

Putting B1's value and solving

$$B_2 = \frac{Pe_2Q}{1 - \{(1-e_2) + (1-P)e_1\}} \tag{4.10}$$

Defective $B2$goods are sold in the market and replaced by customers with new items.Replacement of damaged products increases demand for new items. Genuine market demand D and replacement demand ($\frac{B2}{T}$) are combined to form the effective demand D'.

$$D' = D + \frac{B_2}{T}$$

The buyer's cycle duration for each delivery T is $T = \frac{\ }{D'}$ Changing the value of D'

$$T = \frac{Q}{D + \frac{B_2}{T}} = \frac{QT}{DT + B_2}$$

i.e. $DT + B_2 = Q$

$$T = \frac{Q - B_2}{D}$$

In the preceding solution,

$$T = \left[Q - \frac{Pe_2Q}{1 - \{(1-e_2) + (1-P)e_1\}}\right]\frac{1}{D}$$

**The cost of the Buyer per manufacturing cycle**

During each replenishment cycle, Q goods are delivered to the customer. At the end of each cycle, defective products B2 (Consumers have taken over and are keeping it in their inventory for sale) are sent to the vendor for disposal at a discounted cost. The buyer's inventory holding cost is

HCb = n * (Holding Cost of Q items for T time + Holding Cost of B2 items for T time)

$$HC_b = n * \left(h_b \frac{Q}{2}T + h_b \frac{QB_2}{2}T\right) = \frac{nh_b}{2}(Q + B_2)$$

By substituting values for $B_2$ and T and solving, the buyer's cost is

$$HC_b = \frac{nh_b}{2}\left(Q + \frac{Pe_2Q}{1 - \{(1-e_2) + (1-P)e_1\}}\right) * \frac{(1-P)(1-e_1)Q}{[1 - \{P(1-e_2) + (1-P)e_1\}]D}$$

$$HC_b = \frac{nh_b}{2D}\frac{[1 - \{(1-2e_2) + (1-P)e_1\}](1-P)(1-e_1)}{[1 - \{(1-e_2) + (1-P)e_1\}]^2}Q^2$$

Ordering, shipping, post-sale failure (due to the sale of defective items), and holding are all included in the total cost to the customer for each production cycle. These expenses will be determined by three variable parameters: n (number of orders per manufacturing cycle), Q, and (lot size). Thus, the overall cost per production cycle has been reduced

$$TC_{b\alpha\beta}(n, Q) = K + nF_1 + nc_2(Q + B_{\alpha\beta})pe + \frac{nh_b}{2D}\frac{[1 - \{(1 - 2e_2) + (1 - p)e_1\}](1 - p)(1 - e_1)}{[1 - \{(1 - e_2) + (1 - p)e_1\}]^2}Q^2$$

$$TC_b(n, Q) = K + nF + \frac{nc_{\alpha\beta}pe_2Q}{1 - \{(1 - e_2) + (1 - p)e_1\}} + \frac{nh_b}{2D}\frac{[1 - \{(1 - 2e_2) + (1 - p)e_1\}](1 - p)(1 - e_1)}{[1 - \{(1 - e_2) + (1 - p)e_1\}]^2}Q^2$$

Let

$$A = 1 - \{(1 - e_2) + (1 - p)e_1\}$$

Then,

$$TC_b(n, Q) = K + nF + \frac{nc_{\alpha\beta}pe_2Q}{A} + \frac{nh_b}{2D}\frac{[1 - \{(1 - 2e_2) + (1 - p)e_1\}](1 - p)(1 - e_1)}{A^2}Q^2$$

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