Hop domination number of Tadpole graph $T_{m,n}$

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Abstract

Let $T_{m,n}$ be a tadpole graph. A set $S_h \subseteq V(T_{m,n})$ is a hop dominating set of $T_{m,n}$ if for all v in $V - S_h$, there exists u in S_h such that $d(u,v) = 2$. The minimum cardinality of a hop dominating set of G is called the hop domination number of G and is denoted by $\gamma_h(T_{m,n})$. In this paper, we have discussed about the hop domination number of tadpole graph.

Keywords: hop-domination, hop-domination number, Tadpole graph, neighbourhood.

1. Introduction

[6] The Tadpole graph (Truszczynski 1984) or Kite graph (Kim and park 2006) is the graph obtained by joining a cycle graph to a path graph with a bridge. It is denoted by $T_{m,n}$. The Tadpole graph $T_{m,1}$ is called as m-pan graph and in particular $T_{3,1}$ *and* $T_{4,1}$ are called as paw graph and Banner graph respectively. The generalised Tadpole Graph is

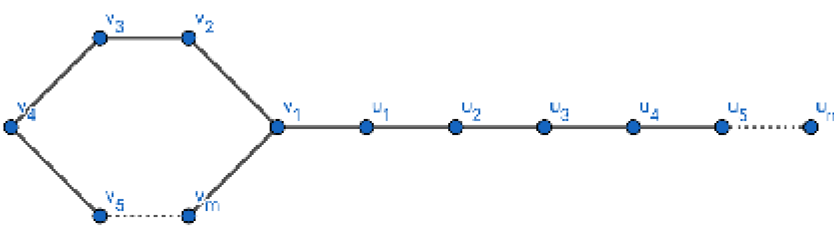


Fig. 1.1

Let us denote the vertices of a Tadpole graph as two distinct sets:

(i) Refer the vertices of the cycle graph C_m *as* $\{v_1, v_2, \dots v_m\}$ and

(ii) The Vertices of the Path graph P_n *as* $\{u_1, u_2, \dots u_n\}$

∴ The vertices of $T_{m,n}$ are

$$V(T_{m,n}) = V(C_m) \cup V(P_n)$$

$$= \{v_1, v_2, \dots v_m, u_1, u_2, \dots u_n\}$$

Theorem 1.1 ([11] p.546): A dominating set D of a graph G is minimal iff for each vertex $v \in D$, one of the following conditions satisfied,

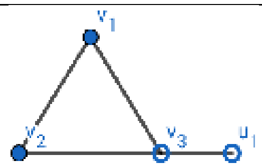
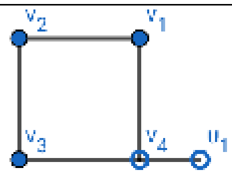
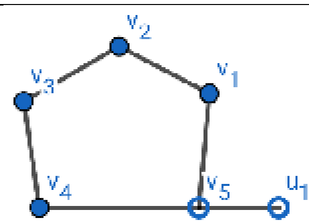
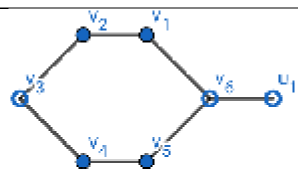
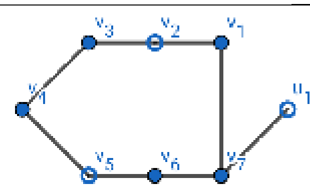
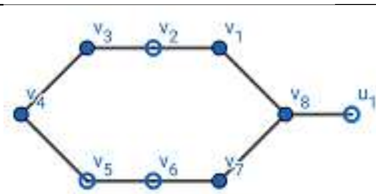
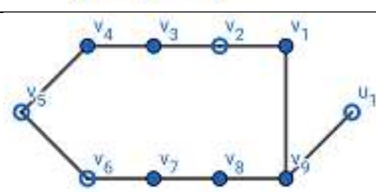
(i) There exists a vertex $u \in V - D$ *such that* $N(u) \cap D = \{v\}$

(ii) v is an isolated vertex in D.

[3] A subset S_h *of* $V(T_{m,n})$ is a hop dominating set of $T_{m,n}$ if for all v in $V - S_h$, there exists u in S_h such that $d(u,v) = 2$. The minimum cardinality of a hop dominating set of G is called the hop domination number of G

and is denoted by $\gamma_h(T_{m,n})$. For any vertex $v \in V(T_{m,n})$, the open neighbourhood of v is the set $N(v) = \{u \in V(T_{m,n}) | uv \in E(T_{m,n})\}$ and the closed neighbourhood is $N[v] = N(v) \cup \{v\}$. For a set $S_h \subseteq V(T_{m,n})$, the open neighbourhood of S_h is $N(S_h) = \cup_{v \in S_h} N(v)$ and the closed neighbourhood is $N[S_h] = N(S_h) \cup S_h$. A set $S_h \subseteq V(T_{m,n})$ is hop dominating set if $N[S_h] = V(T_{m,n})$.

2. Diagrammatic discussion on Hop domination number of Tadpole Graph

S.No.	Pan Graph (P_n)	Graph	$\gamma_h(G)$
1	n=3, P_3		2
2	n=4, P_4		2
3	n=5, P_5		2
4	n=6, P_6		3
5	n=7, P_7		3
6	n=8, P_8		4
7	n=9, P_9		4

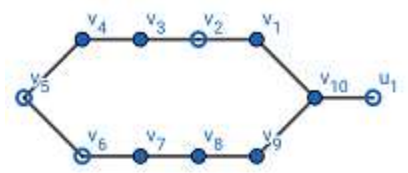
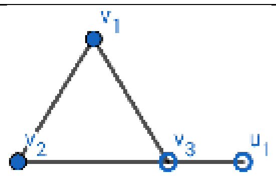
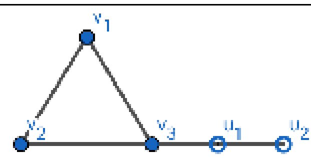
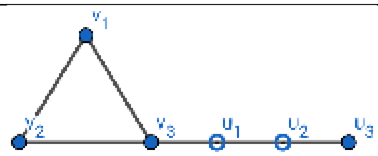
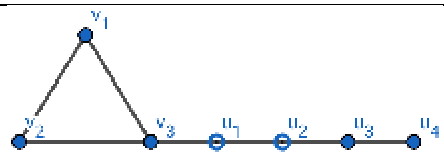
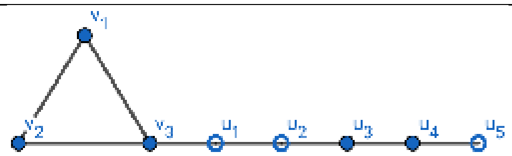
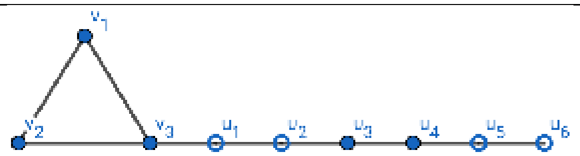
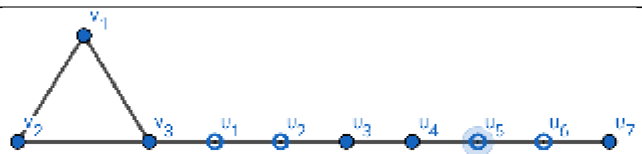
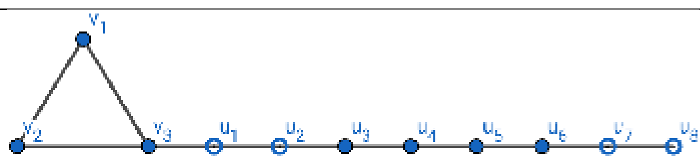
8	n=10, P_{10}		4

Table 2.1: Pan Graph(P_n)

S.No.	Tadpole Graph $(T_{m,n}), m = 3$	Graph	$\gamma_h(G)$
1	n=1, $T_{3,1}$ (paw graph)		2
2	n=2, $T_{3,2}$		2
3	n=3, $T_{3,3}$		2
4	n=4, $T_{3,4}$		2
5	n=5, $T_{3,5}$		3
6	n=6, $T_{3,6}$		4
7	n=7, $T_{3,7}$		4
8	n=8, $T_{3,8}$		4

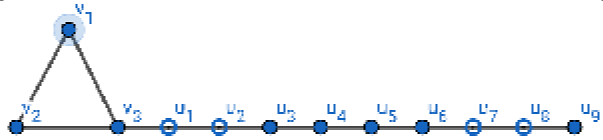
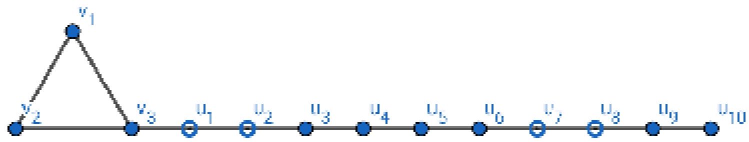
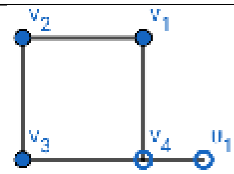
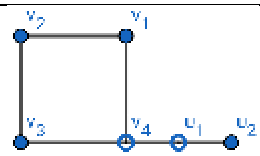
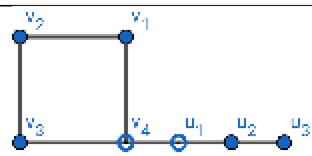
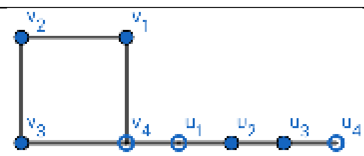
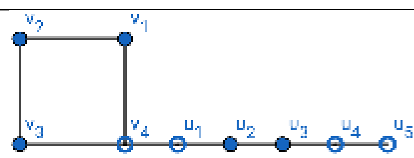
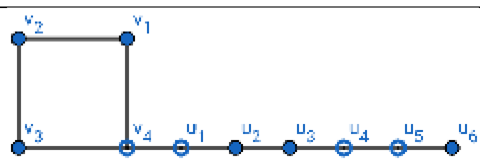
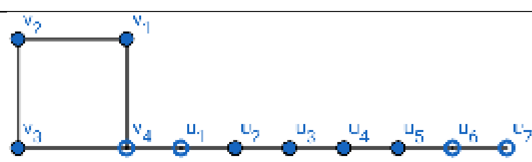
9	n=9, $T_{3,9}$		4
10	n=10, $T_{3,10}$		4

Table 2.2: Tadpole Graph $(T_{m,n}), m = 3$

S.No.	**Tadpole Graph $(T_{m,n}), m = 4$**	**Graph**	$\gamma_h(G)$
1	n=1, $T_{4,1}$ (Banner graph)		2
2	n=2, $T_{4,2}$		2
3	n=3, $T_{4,3}$		2
4	n=4, $T_{4,4}$		3
5	n=5, $T_{4,5}$		4
6	n=6, $T_{4,6}$		4
7	n=7, $T_{4,7}$		4

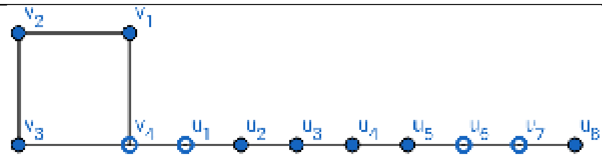
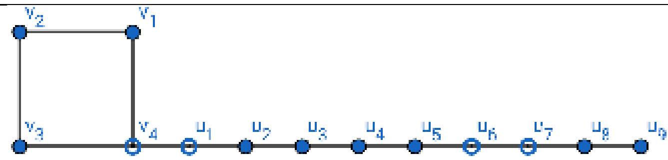
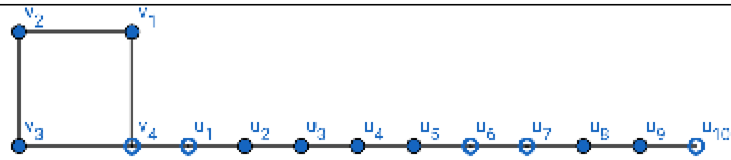
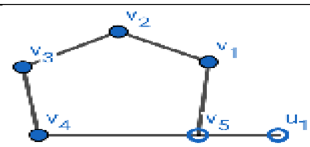
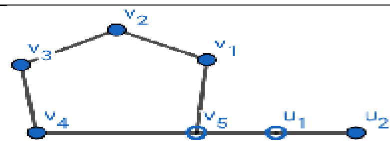
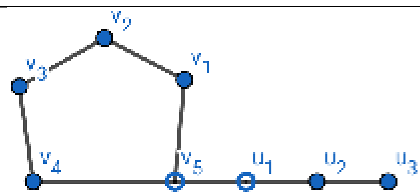
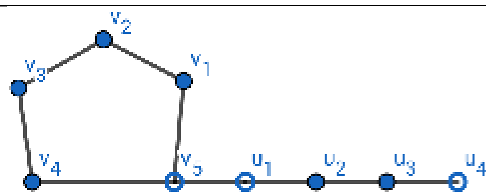
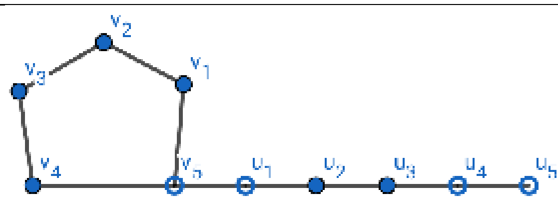
8	n=8, $T_{4,8}$		4
9	n=9, $T_{4,9}$		4
10	n=10, $T_{4,10}$		5

Table 2.3: Tadpole Graph $(T_{m,n}), m = 4$

S. No	**Tadpole Graph $(T_{m,n}), m = 5$**	**Graph**	$\gamma_h(G)$
1	n=1, $T_{5,1}$		2
2	n=2, $T_{5,2}$		2
3	n=3, $T_{5,3}$		2
4	n=4, $T_{5,4}$		3
5	n=5, $T_{5,5}$		4

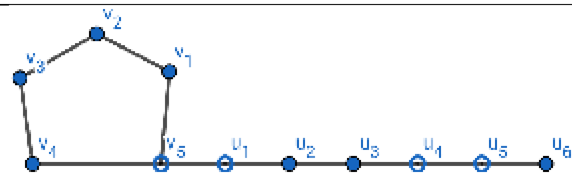
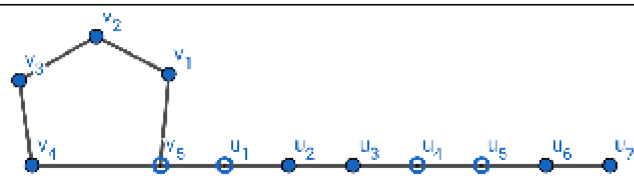
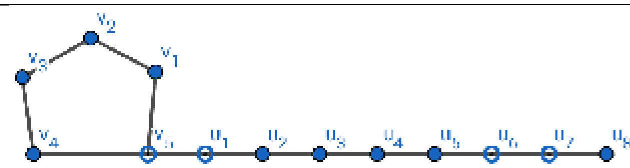
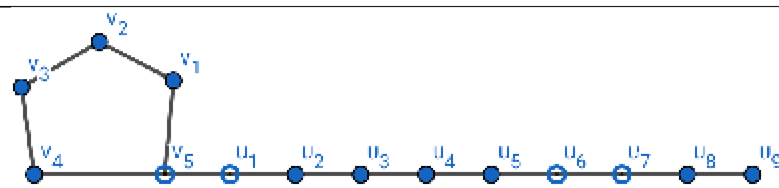
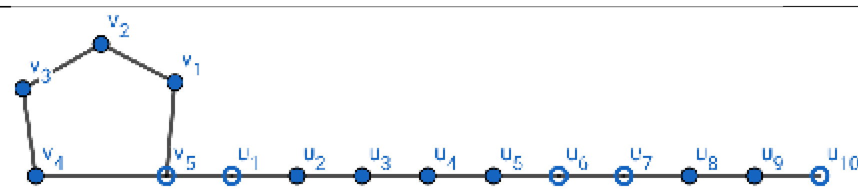
6	n=6, $T_{5,6}$		4
7	n=7, $T_{5,7}$		4
8	n=8, $T_{5,8}$		4
9	n=9, $T_{5,9}$		4
10	n=10, $T_{5,10}$		5

Table 2.4: Tadpole Graph $(T_{m,n}), m = 5$

3. Results on Hop domination number of Tadpole graph $T_{m,n}$

Theorem 3.1: For m-pan graph, the hop domination number is given by

$$\gamma_h = \begin{cases} 2p & iff \quad m = 6p \\ 2p+1 & if \quad m = 6p+1 \\ 2p+2 & if \quad m = 6p+r, 2 \le r \le 5 \end{cases}$$

Proof: Let S_h be the hop dominating set of $T_{m,1}$. The minimality of S_h follows from theorem(1.1) using the contrary of this theorem. If S_h is not a minimal hop dominating set then there exists $v \in S_h$ such that $S_h{}' = S_h - \{v\}$ is a hop dominating set of $T_{m,1}$. Therefore for all $u \in N'[v]$ there exists $v' \in u \in N'[v] - \{v\}, v' \in N'[v]$.

Case(i): If m=6p.

Let $S_h = \{c_{6k-5}, c_{6k-4} | k = 1,2, \dots, p\}$. → (1)

If $v = c_{6k-5}$, then atleast one vertex of $\{p_1, c_{m-1}, c_{6k-3}, c_{6k-7} | k = 1,2, \dots, p\}$ is not hop dominating with any vertex in $S_h{}'$. If $v = c_{6k-4}$, then atleast one vertex of $\{c_m, c_{6k-6}, c_{6k-2} | k = 1,2, \dots, p\}$ is not hop dominated by any vertex in $S_h{}'$.Therefore, $S_h{}'$ is not a hop dominating set. Hence S_h is the minimum. Since for each k, $1 \le k \le p$, there exists c_{6k-5}, c_{6k-4} in $|S_h| = 2p$. $\gamma_h(T_{m,1}) = 2p \; if \; m = 6p$.

Conversely, If $\gamma_h(T_{m,1}) = 2p = |S_h|$, where S_h is given by an equation (1). Hence $V - S_h = \{c_{6k-2}, c_{6k-3}, c_{6k-6}, c_{6k-7}, p_1 \mid k = 1,2, \dots, p\}$. $|V - S_h| = 4p + 1$. We know that $V = (V - S_h) \cup S_h$, therefore $|V| = 4p + 1 + 2p = 6p + 1$. Hence $m = 6p$.

Thus $\gamma_h(T_{m,1}) = 2p\ iff\ m = 6p$.

Case(ii): If m=6p+1.

Let $S_h = \{c_{6k-5}, c_{6k-4}, c_{m-2} | k = 1,2, \dots, p\}$.

If $v = c_{6k-5}\ or\ c_{6k-4}$, the minimality of S_h follows from the above case(i) or else if $v = c_{m-2}$, there is no vertex in ${S_h}'$ hop dominating with c_{m-2}. Hence ${S_h}'$ is not a hop dominating set. Thus S_h is minimum and for each k, $1 \le k \le p$, there exists $c_{6k-5}, c_{6k-4}\ in\ S_h$ and there exists $c_{m-2}\ in\ S_h$ independent of k. Therefore $|S_h| = 2k + 1$. $\gamma_h(T_{m,1}) = 2p + 1\ if\ m = 6p + 1$.

Case(iii): If m=6p+2.

Let $S_h = \{c_{6k-5}, c_{6k-4}, c_{m-2}, c_{m-3} | k = 1,2, \dots, p\}$.

If $v = c_{6k-5}\ or\ c_{6k-4}$, the minimality of S_h follows from the above case(i) or else if $v = c_{m-2}\ or\ c_{m-3}$ there is no vertex in ${S_h}'$ hop dominating with $c_{m-2}\ or\ c_{m-3}$ respectively. Hence ${S_h}'$ is not a hop dominating set. Thus S_h is minimal hop dominating set and for each k, $1 \le k \le p$, there exists $c_{6k-5}, c_{6k-4}\ in\ V - S_h$ and there exists $c_{m-2}, c_{m-3}\ in\ S_h$ independent of k. Therefore $|S_h| = 2k + 2$.

Case(iv): If m=6p+3.

Let $S_h = \{c_{6k-5}, c_{6k-4}, c_{m-2}, c_{m-3} | k = 1,2, \dots, p\}$.

If $v = c_{6k-5}\ or\ c_{6k-4}\ or\ c_{m-2}$, the minimality of S_h follows from the above case(i) and case (iii) or if $v = c_{m-3}$ there is no vertex in ${S_h}'$ hop dominating c_{m-5} in $V - {S_h}'$. Therefore $|S_h| = 2k + 2$.

Case (v): If m=6p+4 and 6p+5.

Let $S_h = \{c_{6k-5}, c_{6k-4} | k = 1,2, \dots, p\}$. The minimality of S_h follows from case(i) and $|S_h| = 2(2p + 1) = 2p + 2$. Hence $\gamma_h(T_{m,1}) = 2p + 2\ if\ m = 6p + r, 2 \le r \le 5$.

Let us indicate the vertices of $T_{m,n}$ as two sets first to refer the vertices of cycle graph C_m as $\{c_1, c_2, \dots, c_m\}$ and the second to refer the vertices of path graph P_n as $\{p_1, p_2, \dots, p_n\}$. So the vertices of $T_{m,n}$ is denoted as $V(T_{m,n}) = \{\{c_1, c_2, \dots, c_m\} \cup \{p_1, p_2, \dots, p_n\}\}$. Let the dominating set of $T_{m,n}$ be S_h.

Theorem 3.2: When m=6p, the hop domination of a tadpole graph $T_{m,n}$ is given by

$$\gamma_h(T_{m,n}) = \begin{cases} 2p + 2k & if \quad n = 6k + r, 0 \le r \le 2 \\ 2p + 2k + 1 & if \quad n = 6k + 3 \\ 2p + 2k + 2 & if \quad n = 6k + r, r = 4,5 \end{cases}$$

Proof: Let S_h be the hop dominating set of $T_{m,1}$. The minimality of S_h follows from theorem(3.1) using the contrary of this theorem. If S_h is not a minimal hop dominating set then there exists $v \in S_h$

such that ${S_h}' = S_h - \{v\}$ is a hop dominating set of $T_{m,1}$. Therefore for all $u \in N'[v]$ there exists $v' \in u \in N'[v] - \{v\}, v' \in N'[v]$.

Case(i): If n=6k

Let $S_h = \{c_{6q-2}, c_{6q-3}, p_{6s-2}, p_{6s-2} | q = 1,2, \dots, m \; and \; s = 1,2, \dots, p\}$

If $v = c_{6q-2}$, then there is no vertex in ${S_h}'$ hop dominating $c_{6q-4} \; and \; c_{6q}$.If $v = c_{6q-3}$, then there is no vertex in ${S_h}'$ hop dominating $c_{6q-5} \; and \; c_{6q-1}$.If $v = c_{6s-2}$, then there is no vertex in ${S_h}'$ hop dominating $c_{6s-4} \; and \; c_{6s}$.If $v = c_{6s-3}$, then there is no vertex in ${S_h}'$ hop dominating $c_{6s-1} \; and \; c_{6s-5}$.

Thus ${S_h}'$ is not minimal hop dominating set. Hence S_h is the minimal hop dominating set.

Case(ii): If n=6k+1

Let $S_h = \{c_{6q-5}, c_{6q-4}, p_{6s-1}, p_{6s-2} | q = 1,2, \dots, m \; and \; s = 1,2, \dots, p\}$

If $v = c_{6q-5}$, there is no vertex in ${S_h}'$ hop dominating c_{6q-3}, c_{6q-7}. In particular, If $q = 1$, there is no vertex in ${S_h}'$ hop dominating $c_{m-1}, c_3 \; and \; p_1$. If $v = c_{6q-4}$, there is no vertex in ${S_h}'$ hop dominating $c_{6q-2} \; and \; c_{6q-6}$. In particular, If $q = 1$, there is no vertex in ${S_h}'$ hop dominating $c_m \; and \; c_4$. If $v = p_{6s-1}$, there is no vertex in ${S_h}'$ hop dominating $p_{6s-3} \; and \; p_{6s+1}$. If $v = p_{6s-2}$, there is no vertex in ${S_h}'$ hop dominating $p_{6s-4} \; and \; p_{6s}$.

Thus ${S_h}'$ is not minimal hop dominating set. Hence S_h is the minimal hop dominating set.

Case(iii): n=6k+2

Let $S_h = \{c_{6q}, c_{6q-1}, p_{6s}, p_{6s-1} | q = 1,2, \dots, m \; and \; s = 1,2, \dots, p\}$

(i)If $v = c_{6q}$, there is no vertex in ${S_h}'$ hop dominating $c_{6q+2} \; and \; c_{6q-2}$. In particular, If $v = c_m$, there is no vertex in ${S_h}'$ hop dominating $c_2, c_{m-2} \; and \; p_2$. (ii)If $v = c_{6q-1}$, there is no vertex in ${S_h}'$ hop dominating $c_{6q+1} \; and \; c_{6q-3}$. In particular, If $v = c_{m-1}$, there is no vertex in ${S_h}'$ hop dominating $c_1, c_{m-3} \; and \; p_1$.(iii) If $v = p_{6s}$, there is no vertex in ${S_h}'$ hop dominating $p_{6s-2} \; and \; p_{6s+2}$. If $v = p_{6s-1}$, there is no vertex in ${S_h}'$ hop dominating $p_{6s-3} \; and \; p_{6s+1}$.

In the above cases (i), (ii) and (iii) for each $q, 1 \le q \le m$, there exists $c_i \; and \; c_{i+1} \; in \; S_h$ and for each $s, 1 \le s \le p$, there exists $p_i \; and \; p_{i+1} \; in \; S_h$, hence $|S_h| = 2p + 2k$.

Thus $\gamma_h(T_{m,n}) = \mathrm{2p + 2k \; if \; m = 6p \; and \; n = 6k + r, 0 \le r \le 2}$.

Case(iv): If n=6k+3.

Let $S_h = \{c_{6q}, c_{6q+1}, p_{6s}, p_{6s+1} | q = 0,1, \dots, m \; and \; s = 0,1,2, \dots, p\}$

(i): If $v = c_1$, there is no vertex in ${S_h}'$ hop dominating c_3.(ii): If $v = c_{6q}$, proof follows from (i) of case(iii). (iii): If $v = c_{6q+1}$, there is no vertex in ${S_h}'$ hop dominating c_{6q-1}, c_{6q+3}. (iv): If $v = p_{6s}$, the proof follows from (iii) of case(iii). (v): If $v = p_{6s+1}$, , there is no vertex in ${S_h}'$ hop dominating p_{6s-1}, p_{6s+3}.

Hence $S_h{}'$ is not minimum. Thus S_h is the hop dominating set and $|S_h| = 2p + 2k + 1$.

Thus $\gamma_h(T_{m,n}) = 2p + 2k + 1$ if m = 6p and n = 6k + 3.

Case(v): If n=6k+4.

Let $S_h = \{c_1, c_2, c_{6q+1}, c_{6q+2}, p_{6s+1}, p_{6s+2} | q = 0,1, \dots, m \; and \; s = 0,1,2, \dots, k\}$

(i): If $v = c_1$, the proof follows from (i) of case(iv). (ii): If $v = c_2$, there is no vertex in $S_h{}'$ hop dominating c_4. (iii): If $v = c_{6q+1}$, the proof follows from (iii) of case(iv). (iv): If $v = c_{6q+2}$, there is no vertex in $S_h{}'$ hop dominating $c_{6q} \; and \; c_{6q+1}$. (v): If $v = p_{6s+1}$, the proof follows from (v) of case(iv). (vi): If $v = p_{6s+2}$, there is no vertex in $S_h{}'$ hop dominating $p_{6s} \; and \; p_{6s+4}$.

Case(vi): If n=6k+5

Let $S_h = \{c_{6q+2}, c_{6q+3}, p_{6s+2}, p_{6s+3} | q = 0,1, \dots, m - 1 \; and \; s = 0,1,2, \dots, k\}$

(i): If $v = c_{6q+2}$, there is no vertex in $S_h{}'$ hop dominating $c_{6q+4} \; and \; c_{6q}$. (ii): If $v = c_{6q+3}$, there is no vertex in $S_h{}'$ hop dominating $c_{6q+5} \; and \; c_{6q+1}$. (iii): If $v = p_{6s+2}$, the proof follows from (iv) of case(v). (iv): If $v = p_{6s+3}$, there is no vertex in $S_h{}'$ hop dominating p_{6s+5}, p_{6s+1}.

In case (v) and (vi), $S_h{}'$ is not minimum. Thus S_h is the hop dominating set and $|S_h| = 2p + 2k + 2 \; if \; m = 6p \; and \; n = 6p + r, r = 4 \; and \; 5$.

Thus $\gamma_h(T_{m,n}) = 2p + 2k + 2 \; if \; m = 6p \; and \; n = 6p + r, r = 4 \; and \; 5$.

Theorem 3.3: When $m = 6p + 1$, the hop domination of a tadpole graph, $T_{m,n}$ is given by,
$$\gamma_h(T_{m,n}) = \begin{cases} 2p + 2k + 1 \; if \; n = 6k + r, \; 0 \le r \le 2 \\ 2p + 2k + 2 \; if \; n = 6k + r, 3 \le r \le 5 \end{cases}$$

Proof: Let S_h be the hop dominating set of $T_{m,1}$. The minimality of S_h follows from theorem(3.1) using the contrary of this theorem. If S_h is not a minimal hop dominating set then there exists $v \in S_h$ such that $S_h{}' = S_h - \{v\}$ is a hop dominating set of $T_{m,1}$. Therefore for all $u \in N'[v]$ there exists $v' \in u \in N'[v] - \{v\}, v' \in N'[v]$.

Case(i): n=6k., Let $S_h = \{c_1, c_{6q-2}, c_{6q-1}, p_{6s-3}, p_{6s-2} | q = 1,2, \dots, p \; \& \; s = 1,2, \dots, k\}$

If $v = c_1$, there is no vertex in $S_h{}'$ hop dominating c_1. If $v = c_{6q-2} \; or \; c_{6q-1} \; or \; p_{6s-3} \; or \; p_{6s-2}$, the proof follows from case(i) of theorem (3.3). Thus $S_h{}'$ is not minimal.

Case(ii): n=6k+1. Let $S_h = \{c_2, c_{6q-1}, c_{6q}, p_{6s-1}, p_{6s-2} | q = 1,2, \dots, p \; \& \; s = 1,2, \dots, k\}$

If $v = c_2$, there is no vertex in $S_h{}'$ hop dominating c_2. If $v = c_{6q} \; or \; c_{6q-1}$, the proof follows from case(iii) of theorem (3.3). If $v = p_{6s-3} \; or \; p_{6s-2}$, the proof follows from case(ii) of theorem (3.3).

Therefore, $S_h{}'$ is not minimal.

Case(iii): n= 6k+2. Let $S_h = \{c_1, c_{6q}, c_{6q+1}, p_{6s}, p_{6s-1} | q = 0,1,2, \dots, p \; \& \; s = 1,2, \dots, k\}$.

If $v = c_1 \ or \ c_{6q} \ or \ c_{6q+1} \ or \ p_{6s}$, the proof follows from case(iv) of theorem (3.3).If $v = p_{6s-1}$, the proof follows from subcase (iv) of case(iii) of theorem (3.3).

Therefore, $S_h{}'$ is not minimal. Hence S_h is hop dominating set and $|S_h| = 2p + 2k + 1 \ if \ n = 6k + r, 0 \le r \le 2$.

Case(iv): n=6k+3. Let $S_h = \{c_3, c_4, c_{6q-4}, c_{6q-5}, p_{6s}, p_{6s+1} | q = 2, \dots, p+1 \ \& \ s = 0,1,2, \dots, k\}$.

If $v = c_3 \ or \ c_4$, the proof follows from case(v) of theorem (3.3). If $v = c_{6q-4} \ or \ c_{6q-5}$, the proof follows from case(iii) of theorem (3.3).If $v = p_{6s} \ or \ p_{6s+1}$, the proof follows from case(iv) of theorem (3.3).

Case(v): n=6k+4. Let $S_h = \{c_{6q-3}, c_{6q-4}, p_{6s+1}, p_{6s+2} | q = 1,2, \dots, p \ \& \ s = 0,1,2, \dots, k\}$.

If $v = c_{6q-3}$, the minimality of D follows from case(i) of theorem (3.3) or else if $v = c_{6q-4}$, it follows from case (ii) of theorem(3.3). If $v = p_{6s+1} \ or \ p_{6s+2}$, the minimality of S_h follows from case(iv) of theorem(3).

Case(vi): n=6k+5. Let $S_h = \{c_{6q-2}, c_{6q-3}, p_{6s+2}, p_{6s+3} | q = 1,2, \dots, p \ \& \ s = 0,1,2, \dots, k\}$.

If $v = c_{6q-2} \ or \ c_{6q-3}$, the minimality of S_h follows from case(i) of theorem(3.3). If $v = p_{6s+2} \ or \ p_{6s+3}$, the minimality of S_h follows from case(vi) of theorem (3.3).

Hence S_h is the hop dominating set and $|S_h| = 2p + 2k + 2 \ if \ n = 6k + r, 3 \le r \le 5$. Thus when m=6p+1, $\gamma_h(T_{m,n}) = \begin{cases} 2p + 2k + 1 \ if \ n = 6k + r, \ 0 \le r \le 2 \\ 2p + 2k + 2 \ if \ n = 6k + r, 3 \le r \le 5. \end{cases}$

Theorem 3.4: When $m = 6p + 2$, the hop domination of $T_{m,n}$ is given

$$\gamma_h(T_{m,n}) = \begin{cases} 2p + 2k + 2 & if \ n = 6k + r, 0 \le r \le 4 \\ 2p + 2k + 3 & if \ n = 6k + 5 \end{cases}$$

Proof: Let S_h be the hop dominating set of $T_{m,1}$. The minimality of S_h follows from theorem(3.1) using the contrary of this theorem. If S_h is not a minimal hop dominating set then there exists $v \in S_h$ such that $S_h{}' = S_h - \{v\}$ is a hop dominating set of $T_{m,1}$. Therefore for all $u \in N'[v]$ there exists $v' \in u \in N'[v] - \{v\}, v' \in N'[v]$.

Case (i): If $n = 6k$, $S_h = \{c_1, c_2, c_{6q-1}, c_{6q}, p_{6s-3}, p_{6s-2} \ / \ q = 1,2 \dots \dots, p \ \& \ s = 1,2, \dots \dots, k\}$

Case (ii): If $n = 6k + 1$, $S_h = \{c_2, c_3, c_{6q}, c_{6q+1}, p_{6s-2}, p_{6s-1} \ / \ q = 1,2 \dots \dots, p \ \& \ s = 1,2, \dots \dots, k\}$

Case (iii): If $n = 6k + 2$, $S_h = \{c_3, c_4, c_{6q+1}, c_{6q+2}, p_{6s-1}, p_{6s} \ / \ q = 1,2 \dots \dots, p \ \& \ s = 1,2, \dots \dots, k\}$

Case (iv): If $n = 6k + 3$, $S_h = \{c_{6q+2}, c_{6q+3}, p_{6s}, p_{6s+1} \ / \ q = 0,1, \dots \dots, p \ \& \ s = 0,1, \dots \dots, k\}$

Case (v): If $n = 6k + 4$, $S_h = \{c_{6q-3}, c_{6q-2}, p_{6s+1}, p_{6s+2} \ / \ q = 1,2 \dots \dots, p \ \& \ s = 0,1,2, \dots \dots, k\}$

When $m = 6p + 2$, the minimality of S_h follows as the previous theorem and $|S_h| = 2p + 2k + 2$ if $n = 6k + r, 0 \le r \le 4$.

Case (vi): $n = 6k + 5, S_h = \{c_1, c_{6q-1}, c_{6q-2}, p_{6s+2}, p_{6s+3} \,/\, q = 1,2 \dots \dots, p \,\&\, s = 0,1,2, \dots \dots, k\}$

When $m = 6p + 2$ and $n = 6k + 5$, the S_h is the hop dominating set as of from the previous theorems and $|S_h| = 2p + 2k + 3$.

Thus when $= 6p + 2$,$\gamma_h(T_{m,n}) = \begin{cases} 2p + 2k + 2 & if\ n = 6k + r, 0 \le r \le 4 \\ 2p + 2k + 3 & if\ n = 6k + 5. \end{cases}$

Theorem 3.5: When $m = 6p + 3$, the hop domination of $T_{m,n}$ is given

$$\gamma_h(T_{m,n}) = \begin{cases} 2p + 2k + 2 & if\ n = 6k + r, 0 \le r \le 4 \\ 2p + 2k + 4 & if\ n = 6k + 5 \end{cases}$$

Proof:

Let S_h be the hop dominating set of $T_{m,1}$. The minimality of S_h follows from theorem(3.1) using the contrary of this theorem. If S_h is not a minimal hop dominating set then there exists $v \in S_h$ such that $S_h{}' = S_h - \{v\}$ is a hop dominating set of $T_{m,1}$. Therefore for all $u \in N'[v]$ there exists $v' \in u \in N'[v] - \{v\}, v' \in N'[v]$.

Case (i): If $n = 6p, S_h = \{c_2, c_3, c_{6q}, c_{6q+1}, p_{6s-3}, p_{6s-2} \,/\, q = 1,2 \dots \dots, p \,\&\, s = 1,2, \dots \dots, k\}$

Case (ii): If $n = 6p + 1, S_h = \{c_{6q+1}, c_{6q+2}, p_{6s-2}, p_{6s-1} \,/\, q = 0,1,2 \dots \dots, p \,\&\, s = 1,2, \dots \dots, k\}$

Case (iii): If $n = 6p + 2, S_h = \{c_{6q+2}, c_{6q+3}, p_{6s-1}, p_{6s} \,/\, q = 0,1,2 \dots \dots, p \,\&\, s = 1,2, \dots \dots, k\}$

Case (iv): If $n = 6p + 3, S_h = \{c_{6q-3}, c_{6q-2}, p_{6s}, p_{6s+1} \,/\, q = 1,2 \dots \dots, p \,\&\, s = 0,1, \dots \dots, k\}$

Case (v): If $n = 6p + 4, S_h = \{c_{6q-2}, c_{6q-1}, p_{6s+1}, p_{6s+2} \,/\, q = 1,2 \dots \dots, p \,\&\, s = 0,1,2, \dots \dots, k\}$

Case (vi): If $n = 6p + 5, S_h = \{c_1, c_2, c_{6q-1}, c_{6q}, p_{6s+2}, p_{6s+3} \,/\, q = 1,2 \dots \dots, p \,\&\, s = 0,1,2, \dots \dots, k\}$

When $m = 6p + 3$, the minimality of S_h follows as theorem (2) & (3) and $|S_h| = \gamma_h(T_{m,n}) = \begin{cases} 2p + 2k + 2 & if\ n = 6k + r, 0 \le r \le 4 \\ 2p + 2k + 4 & if\ n = 6k + 5. \end{cases}$

Theorem 3.6: When $m = 6p + 4$, the hop domination of a tadpole graph $T_{m,n}$ is given by

$$\gamma_h(T_{m,n}) = \begin{cases} 2p + 2k + 2 & if\ n = 6k + r, 0 \le r \le 3 \\ 2p + 2k + 3 & if\ n = 6k + 4 \\ 2p + 2k + 4 & if\ n = 6k + 5 \end{cases}$$

Proof: Let S_h be the hop dominating set of $T_{m,1}$. The minimality of S_h follows from theorem(3.1) using the contrary of this theorem. If S_h is not a minimal hop dominating set then there exists $v \in S_h$ such that $S_h{}' = S_h - \{v\}$ is a hop dominating set of $T_{m,1}$. Therefore for all $u \in N'[v]$ there exists $v' \in u \in N'[v] - \{v\}, v' \in N'[v]$.

Case (i): If $n = 6p, S_h = \{c_{6q+1}, c_{6q+2}, p_{6s-3}, p_{6s-2} \,/\, q = 0,1,2 \dots \dots, p \,\&\, s = 1,2, \dots \dots, k\}$

Case (ii): If $n = 6p + 1$, $S_h = \{c_{6q+2}, c_{6q+3}, p_{6s-2}, p_{6s-1} \,/\, q = 0,1,2 \dots \dots, p \;\&\; s = 1,2, \dots \dots, k\}$

Case (iii): If $n = 6p + 2$, $S_h = \{c_{6q+3}, c_{6q+4}, p_{6s-1}, p_{6s} \,/\, q = 0,1,2 \dots \dots, p \;\&\; s = 1,2, \dots \dots, k\}$

Case (iv): If $n = 6p + 3$, $S_h = \{c_{6q+4}, c_{6q+5}, p_{6s}, p_{6s+1} \,/\, q = 0,1,2 \dots \dots, p \;\&\; s = 1,2, \dots \dots, k\}$

Case (v): If $n = 6p + 4$, $S_h = \{c_2, c_{6q-1}, c_{6q}, p_{6s+1}, p_{6s+2} \,/\, q = 1,2 \dots \dots, p \;\&\; s = 0,1,2, \dots \dots, k\}$

Case (vi): If $n = 6p + 5$, $S_h = \{c_1, c_2, c_{6q}, c_{6q+1}, p_{6s+1}, p_{6s+2} \,/\, q = 1,2 \dots \dots, p \;\&\; s = 0,1, \dots \dots, k\}$

When m=6p+4, the minimality of S_h follows as in theorem (3.2) & (3.3) and

$$|S_h| = \gamma_h(T_{m,n}) = \begin{cases} 2p + 2k + 2 \; if \; n = 6k + r, 0 \le r \le 3 \\ 2p + 2k + 3 \qquad if \; n = 6k + 4 \\ 2p + 2k + 4 \qquad if \; n = 6k + 5. \end{cases}$$

Theorem 3.7: When $m = 6p + 5$, the hop domination of $T_{m,n}$ is given by

$$\gamma_h(T_{m,n}) = \begin{cases} 2p + 2k + 2 \quad if \; n = 6k + r, 0 \le r \le 3 \\ 2p + 2k + 4 \quad if \; n = 6k + r, r = 4 \; and \; 5 \end{cases}$$

Proof: Let S_h be the hop dominating set of $T_{m,1}$. The minimality of S_h follows from theorem(3.1) using the contrary of this theorem. If S_h is not a minimal hop dominating set then there exists $v \in S_h$ such that $S_h{}' = S_h - \{v\}$ is a hop dominating set of $T_{m,1}$. Therefore for all $u \in N'[v]$ there exists $v' \in u \in N'[v] - \{v\}, v' \in N'[v]$.

Case (i): If $n = 6p$, $S_h = \{c_{6q+2}, c_{6q+3}, p_{6s-3}, p_{6s-2} \,/\, q = 0,1,2 \dots \dots, p \;\&\; s = 1,2, \dots \dots, k\}$

Case (ii): If $n = 6p + 1$, $S_h = \{c_{6q+3}, c_{6q+4}, p_{6s-2}, p_{6s-1} \,/\, q = 0,1,2 \dots \dots, p \;\&\; s = 1,2 \dots \dots, k\}$

Case (iii): If $n = 6p + 2$, $S_h = \{c_{6q+4}, c_{6q+5}, p_{6s-1}, p_{6s} \,/\, q = 0,1,2 \dots \dots, p \;\&\; s = 1,2 \dots \dots, k\}$

Case (iv): If $n = 6p + 3$, $S_h = \{c_{6q}, c_{6q-1}, p_{6s}, p_{6s+1} \,/\, q = 1,2 \dots \dots, p \;\&\; s = 0,1,2 \dots \dots, k\}$

Case (v): If $n = 6p + 4$, $S_h = \{c_2, c_3, c_{6q}, c_{6q+1}, p_{6s+1}, p_{6s+2} \,/\, q = 1,2 \dots \dots, p \;\&\; s = 0,1,2 \dots \dots, k\}$

Case (vi): If $n = 6p + 5$, $S_h = \{c_{6q+3}, c_{6q+4}, p_{6s+2}, p_{6s+3} \,/\, q = 0,1,2 \dots \dots, p \;\&\; s = 0,1,2 \dots \dots, k\}$

When $m = 6p + 5$, the minimality of S_h follows as in theorem (2) and (3), Hence $|S_h| = \gamma_h(T_{m,n}) = \begin{cases} 2p + 2k + 2 \quad if \; n = 6k + r, 0 \le r \le 3 \\ 2p + 2k + 4 \quad if \; n = 6k + r, r = 4 \; and \; 5. \end{cases}$

Theorem 3.8: For $n = 2$ the hop domination of a Tadpole graph $T_{m,n}$ is given by

$$\gamma_h(T_{m,2}) = \begin{cases} 2p \qquad\qquad if \; m = 6p \\ 2p + 1 \qquad if \; m = 6p + 1 \\ 2p + 2 \quad if \; m = 6p + r, 2 \le r \le 5 \end{cases}$$

Proof: Let S_h be the hop dominating set of $T_{m,1}$. The minimality of S_h follows from theorem(3.1) using the contrary of this theorem. If S_h is not a minimal hop dominating set then there exists $v \in S_h$ such that $S_h{}' = S_h - \{v\}$ is a hop dominating set of $T_{m,1}$. Therefore for all $u \in N'[v]$ there exists $v' \in u \in N'[v] - \{v\}, v' \in N'[v]$.

Case (i): $m = 6p$, Let $S_h = \{c_{6q}, c_{6q-1} / q = 1,2, \dots k\}$

(a) If $v = c_{6q}$, $\exists$ no vertex in $S_h{}'$ hop dominating with at least one of the vertex of $\{c_2, p_2, c_{6q+2}, c_{6q-2}\}$.
(b) If $v = c_{6q-1}$, $\exists$ no vertex in $S_h{}'$ hop dominating with at least one of the vertex of $\{c_1, p_1, c_{6q+1}, c_{6q-3}\}$

Thus $S_h{}'$ is not minimum. Hence S_h is the hop dominating set and $|S_h| = 2p$.

Case (ii): $m = 6p + 1$, Let $S_h = \{c_3, c_{6q}, c_{6q+1} / q = 1,2, \dots k\}$

(a) If $v = c_3$, $\exists$ no vertex in $S_h{}'$ hop dominating c_3. (b) If $v = c_{6q}$, $\exists$ no vertex in $S_h{}'$ hop dominating with at least one of the vertex of $\{c_1, p_1, c_{6q+2}, c_{6q-2}\}$. (c) If $v = c_{6q+1}$, $\exists$ no vertex in $S_h{}'$ hop dominating with at least one of the vertex of $\{c_2, p_2, c_{6q+3}, c_{6q-1}\}$.

Thus S_h is the hop dominating set and $|S_h| = 2p + 1$.

Case (iii): $m = 6p + 2$, Let $S_h = \{c_{6q+1}, c_{6q+2} / q = 0,1, \dots k\}$.

(a) If $v = c_{6q+2}$, $\exists$ no vertex in $S_h{}'$ hop dominating with at least one of the vertex of $\{c_{6q+1}, c_{6q-1} \; or \; p_1\}$.
(b) If $v = c_{6q+1}$, $\exists$ no vertex in $S_h{}'$ hop dominating with at least one of the vertex of $\{c_{6q}, c_{6q+4} \; or \; p_2\}$

Thus S_h is the hop dominating set and $|S_h| = 2p + 2$

Case (iv): $m = 6p + 3$, Let $S_h = \{c_{6q+2}, c_{6q+3} / q = 0,1, \dots k\}$

(a) If $v = c_{6q+2}$, $\exists$ no vertex in $S_h{}'$ hop dominating with at least one of the vertex of $\{c_{6q}, c_{6q+4}, p_1\}$.
(b)If $v = c_{6q+3}$, $\exists$ no vertex in $S_h{}'$ hop dominating with at least one of the vertex of $\{c_{6q-1}, c_{6q+5}, p_2\}$.

Thus S_h is the hop dominating set and $|S_h| = 2p + 2$

Case (v): $m = 6p + 4$, Let $S_h = \{c_{6q+3}, c_{6q+4} / q = 0,1, \dots k\}$

(a) If $v = c_{6q+3}$, $\exists$ no vertex in $S_h{}'$ hop dominating with at least one of the vertex of $\{c_{6q-1}, c_{6q+5}, p_1\}$.
(b) If $v = c_{6q+4}$, $\exists$ no vertex in D' hop dominating with at least one of the vertex of $\{c_{6q-2}, c_{6q+6}, p_2\}$.

Thus S_h is the hop dominating set and $|S_h| = 2p + 2$.

Case (vi): $m = 6p + 5$, Let $S_h = \left\{c_{6q+4}, \frac{c_{6q+5}}{} q = 0,1, \dots k\right\}$

(a) If $v = c_{6q+4}$, $\exists$ no vertex in $S_h{}'$ hop dominating with at least one of the vertex of $\{c_{6q-2}, c_{6q+6}, p_1\}$.
(b) If $v = c_{6q+5}$, $\exists$ no vertex in $S_h{}'$ hop dominating with at least one of the vertex of $\{c_{6q-3}, c_{6q+7}, p_2\}$.

Thus S_h is the hop dominating set and $|S_h| = 2p + 2$

$$\text{Thus, } \gamma_h(T_{m,2}) = \begin{cases} 2p & if \; m = 6p \\ 2p + 1 & if \; m = 6p + 1 \\ 2p + 2 & if \; m = 6p + r, 2 \le r \le 5 \end{cases}$$

Theorem 3.9: For $n = 3$, the hop domination of a Tadpole graph $T_{m,n}$ is given by

$$\gamma_h(T_{m,3}) = \begin{cases} 2p+1 & if\ m = 6p \\ 2p+2\ if\ m = 6p+r, 1 \le r \le 5 \end{cases}$$

Proof: Let S_h be the hop dominating set of $T_{m,1}$. The minimality of S_h follows from theorem(3.1) using the contrary of this theorem. If S_h is not a minimal hop dominating set then there exists $v \in S_h$ such that ${S_h}' = S_h - \{v\}$ is a hop dominating set of $T_{m,1}$. Therefore for all $u \in N'[v]$ there exists $v' \in u \in N'[v] - \{v\}, v' \in N'[v]$.

Case (i): $m = 6p$, Let $S_h = \{c_{6q}, c_{6q+1}, p_1 / q = 0,1,2, \ldots k\}$

(a) If $v = c_{6q}$, ∃ no vertex in ${S_h}'$ hop dominating with atleast one of the vertices such as of $\{c_2, c_{6q+2}, c_{6q-2}, p_2\}$. (b) If $v = c_{6q+1}$, ∃ no vertex in ${S_h}'$ hop dominating with atleast one of the vertices such as of $\{c_{6q-3}\ and\ c_{6q-1}\}$. (c) If $v = p_3$, ∃ no vertex in ${S_h}'$ hop dominating with at least one of the vertices such as of $\{c_1, c_{m-1}\ and\ p_3\}$

Thus S_h is the minimal hop dominating set and $|S_h| = 2p + 1$

Case (ii): $m = 6p + 1$, Let $S_h = \{c_{6q}, c_{6q+1}, p_1 / q = 0,2, \ldots k\}$

(a) If $v = c_{6q+1}$, ∃ no vertex in ${S_h}'$ hop dominating with at least one of the vertex such as of $\{c_{6q-1}, c_{6q+1}, c_2, p_2\}$. (b) If $v = c_{6q+2}$, ∃ no vertex in ${S_h}'$ hop dominating with atleast one of the vertices such as of $\{c_{6q}\ and\ c_{6q+4}\}$. (c) If $v = p_1$, ∃ no vertex in ${S_h}'$ hop dominating with atleast one of the vertices such as of $\{c_1, c_{m-1}\ and\ p_3\}$

Thus S_h is the minimal hop dominating set and $|D| = 2p + 2$

Case (iii): $m = 6p + 2$, Let $S_h = \{c_{6q+2}, c_{6q+3}, p_1 / q = 0,1,2, \ldots k\}$

Case (iv): $m = 6p + 3$, Let $S_h = \{c_{6q+3}, c_{6q+4}, p_1 / q = 0,1,2, \ldots k\}$

Case (v): $m = 6p + 4$, Let $S_h = \{c_{6q+4}, c_{6q+5}, p_1 / q = 0,1,2, \ldots k\}$

Case (vi): $m = 6p + 5$, Let $S_h = \{c_{6q}, c_{6q+1}, p_1 / q = 0,1,2, \ldots k\}$

The Proof of case (iii) to (vi) follows same as previous cases. Thus S_h is the minimal hop-dominating set and $|S_h| = 2p + 2$

Thus for $n = 3$, $\gamma_h(T_{m,3}) = \begin{cases} 2p+1 & if\ m = 6p \\ 2p+2\ if\ m = 6p+r, 1 \le r \le 5 \end{cases}$

Theorem 3.10: For $n = 4$, the hop domination of a Tadpole graph $T_{m,n}$ is given by

$$\gamma_h(T_{m,4}) = \begin{cases} 2p+2 & if\ m = 6p+r, 0 \le r \le 3 \\ 2p+3 & if\ m = 6p+4 \\ 2p+4 & if\ m = 6p+5 \end{cases}$$

Proof: Let S_h be the hop dominating set of $T_{m,1}$. The minimality of S_h follows from theorem(3.1) using the contrary of this theorem. If S_h is not a minimal hop dominating set then there exists $v \in S_h$ such that ${S_h}' = S_h - \{v\}$ is a hop dominating set of $T_{m,1}$. Therefore for all $u \in N'[v]$ there exists $v' \in u \in N'[v] - \{v\}, v' \in N'[v]$.

Case (i): $m = 6p$, Let $S_h = \{c_{6q+1}, c_{6q+2}, p_1, p_2 / q = 0,1, \dots k-1\}$

(a) If $v = c_{6q+1}$, ∃ no vertex in ${S_h}'$ hop dominating $\{c_{6q-1}\ and\ c_{6q+3}\}$. (b) If $v = c_{6q+2}$, ∃ no vertex in ${S_h}'$ hop dominating $\{c_{6q}\ and\ c_{6q+4}\}$. (c) If $v = p_1$, ∃ no vertex in ${S_h}'$ hop dominating with $\{p_3\}$. (d) If $v = p_2$, ∃ no vertex in ${S_h}'$ hop dominating with $\{p_4\}$

Thus S_h is the minimal hop dominating set and $|S_h| = 2k + 2$

Case (ii): $m = 6p + 1$, Let $S_h = \{c_{6q+4}, c_{6q+5}, p_1, p_2 / q = 0,1, \dots k-1\}$

The Proof is similar to case (i) and $|D| = 2p + 2$

Case (iii): $m = 6p + 2$, Let $S_h = \{c_{6q+3}, c_{6q+4}, p_1, p_2 / q = 0,1,2, \dots k-1\}$

The Proof is similar to case (i) and $|S_h| = 2p + 2$

Case (iv): $m = 6p + 3$, Let $S_h = \{c_{6q+4}, c_{6q+5}, p_1, p_2 / q = 0,1,2, \dots k-1\}$

The Proof is similar to case (i) and $|S_h| = 2p + 2$

Case (v): $m = 6p + 4$, Let $S_h = \{c_2, c_{6q-1}, c_{6q}, p_1, p_2 / q = 1,2, \dots k\}$

(a) If $v = c_2$, ∃ no vertex in ${S_h}'$ hop dominating $\{c_2\}$. (b) If $v = c_{6q-1}$, ∃ no vertex in ${S_h}'$ hop dominating $\{c_{6q-3}\ and\ c_{6q+1}\}$. (c) If $v = c_{6q}$, ∃ no vertex in ${S_h}'$ hop dominating $\{c_{6q-2}\ and\ c_{6q+2}\}$. (d) If $v = p_1$, ∃ no vertex in ${S_h}'$ hop dominating with $\{p_3\}$. (e) If $v = p_2$, ∃ no vertex in ${S_h}'$ hop dominating with $\{p_4\}$

Thus S_h is the minimal hop dominating set and $|S_h| = 2p + 3$

Case (vi): $m = 6p + 5$, Let $S_h = \{c_2, c_3, c_{6q}, c_{6q+1}, p_1, p_2 / q = 1,2, \dots k\}$

(a) If $v = c_2$, ∃ no vertex in ${S_h}'$ hop dominating $\{c_2\}$. (b) If $v = c_3$, ∃ no vertex in ${S_h}'$ hop dominating $\{c_3\}$. (c) If $v = c_{6q}$, ∃ no vertex in ${S_h}'$ hop dominating $\{c_{6q-2}\ and\ c_{6q+2}\}$. (d) If $v = c_{6q+1}$, ∃ no vertex in ${S_h}'$ hop dominating $\{c_{6q-1}\ and\ c_{6q+3}\}$. (e) If $v = p_1$, ∃ no vertex in ${S_h}'$ hop dominating with $\{p_3\}$. (f) If $v = p_2$, ∃ no vertex in ${S_h}'$ hop dominating with $\{p_4\}$

Thus S_h is the minimal hop dominating set and $|S_h| = 2p + 4$

Hence when $n = 4$, $\gamma_h(T_{m,4}) = \begin{cases} 2p+2 & if\ m = 6p + r, 0 \le r \le 3 \\ 2p+3 & if\ m = 6p+4 \\ 2p+4 & if\ m = 6p+5 \end{cases}$

Theorem 3.11: For $n = 5$, the hop domination of a Tadpole graph $T_{m,n}$ is given by

$$\gamma_h(T_{m,4}) = \begin{cases} 2p+2 & if\ m = 6p + r, r = 0,1 \\ 2p+3 & if\ m = 6p+2 \\ 2p+5 & if\ m = 6p + r, 3 \le r \le 5 \end{cases}$$

Proof: Let S_h be the hop dominating set of $T_{m,1}$. The minimality of S_h follows from theorem(3.1) using the contrary of this theorem. If S_h is not a minimal hop dominating set then there exists $v \in S_h$ such that $S_h{}' = S_h - \{v\}$ is a hop dominating set of $T_{m,1}$. Therefore for all $u \in N'[v]$ there exists $v' \in u \in N'[v] - \{v\}, v' \in N'[v]$.

Case (i): If $m = 6p$, Let $S_h = \{c_{6q+2}, c_{6q+3}, p_2, p_3 \,/\, q = 0,1, \ldots k-1\}$

Case (ii): If $m = 6p + 1$, Let $S_h = \{c_{6q+3}, c_{6q+4}, p_2, p_3 \,/\, q = 0,1,2, \ldots k-1\}$

Case (iii): If $m = 6p + 2$, Let $S_h = \{c_{6q+4}, c_{6q+5}, p_2, p_3 \,/\, q = 0,1,2, \ldots k-1\}$

Case (iv): If $m = 6p + 3$, Let $S_h = \{c_3, c_{6q-1}, c_{6q}, p_2, p_3 \,/\, q = 1,2, \ldots k\}$

Case (v): If $m = 6p + 4$, Let $S_h = \{c_2, c_3, c_{6q}, c_{6q+1}, p_2, p_3 \,/\, q = 1,2, \ldots k\}$

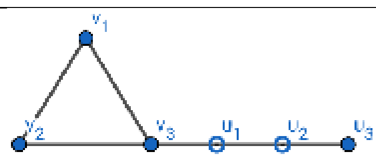
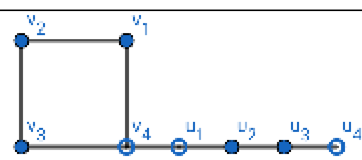
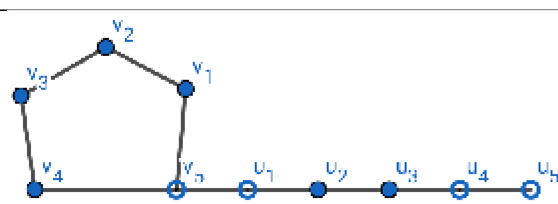
Case (vi): If $m = 6p + 5$, Let $S_h = \{c_{6q+1}, c_{6q+2}, p_2, p_3 \,/\, q = 0,1, \ldots k\}$. The proof follows as the previous theorem.

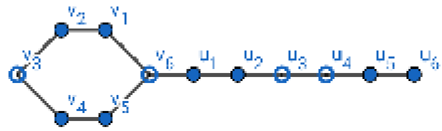
Theorem 3.12: $\gamma_h(T_{m,n}) = \gamma_h(T_{n,m}) \; iff \; m = n.$

Proof: Let $m = n$, then $T_{m,n}$ and $T_{n,m}$ are same graphs. Hence $\gamma_h(T_{m,n}) = \gamma_h(T_{n,m})$. On the other hand, assume $\gamma_h(T_{m,n}) = \gamma_h(T_{n,m})$, suppose if $m \neq n$, then by theorems(--) $\gamma_h(T_{m,n}) \neq \gamma_h(T_{n,m})$. Thus $\gamma_h(T_{m,n}) = \gamma_h(T_{n,m}) \; iff \; m = n$.

Corollary:$\gamma_h(T_{m,n}) = \gamma_h(T_{n,m}) = m - 1 \; (or) \; n - 1 \; iff \; m = n, where \; m = n = 3,4,5.$ and $\gamma_h(T_{m,n}) = \gamma_h(T_{n,m}) \leq m - 1 \; (or) \; n - 1 \; iff \; m = n, where \; m = n = 6.$

Proof:

S.No.	Tadpole Graph $(T_{m,n}), m = n$	Graph	$\gamma_h(G)$
1	m=n=3, $T_{3,3}$		2 or (n-1)
2	m=n=4, $T_{4,4}$		3 or (n-1)
3	m=n=5, $T_{5,5}$		4 or (n-1)

4	m=n=6, $T_{6,6}$		4 or ≤(n-1)

From the table we get $\gamma_h(T_{m,n}) = \gamma_h(T_{n,m}) = m - 1\ (or)\ n - 1\ iff\ \ m = n,$ where

$m = n = 3,4,5$. And $\gamma_h(T_{m,n}) = \gamma_h(T_{n,m}) \leq m - 1\ (or)\ n - 1\ iff\ \ m = n, where\ m = n = 6.$

4. Conclusion: In this paper, we have found the hop domination number of Tadpole graph and derived some theorems on it.

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