A STUDY OF APPLICATIONS OF PROBABILITY, STATISTICS TOWARDS STOCHASTIC PROCESS

C. Subet Selin[1] , Dr. K.Chithamparathanu Pillai**[2] and Dr. M.Anto***[3]*

*[*1]. C.Subet Selin, Ph.D Research Scholar, part-time, Reg.no.19123012092010, Department of mathematics, Annai Velankanni College Tholayavattam, Affiliated to Manonmaniam Sundaranar University, Abishekapatti,Tirunelveli,Tamil Nadu India*

*[**2] Dr. K.Chithamparathanu Pillai, Supervisor, Associate Professor, Department of Mathematics, Lekshmipuram College of Arts and Science, Neyyoor, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli , Tamilnadu, India.*

*[***3] Dr. M.Anto, Co-Supervisor, Associate Professor, Department of Mathematics, Annai Velankanni College, Tholayavattam, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli, Tamilnadu, India.*

ABSTRACT

Probabilistic models for random numbers developed in time or space are stochastic processes. The evolution is driven by some relationship of dependency between random quantities at different times or places. Random walks, Markov processes, ramming processes, processes of renewal, martingales and Brownian motion are the main types of stochastic process. Mathematical financing, queuing processes, computer algorithm analysis, economic time series, image analysis , social networks and biomedical modeling are important areas for the implementation of the application. In operational research applications, stochastic process models are extensively used. The theory of probability is intended to provide a mathematical framework for the description, modeling, analysis and resolution of random phenomena and complex systems. Although it originally studied gambling issues, it is likely that applications in financial, computer, engineering, statistical and biological fields have been successful. The "probabilistic method" is an important means for proving existence theorems in discrete mathematics and is essential for analyzing statistical procedures in the fields of mathematics.

KEYWORDS: Applications, Probability, Statistics, Stochastic Process, probabilistic models

INTRODUCTION

Probability and the stochastic processes using applications provide a clear, easy-to - understand approach to probability and stochastic processes, which gives readers a solid basis for their careers. In particular with an emphasis on applications in the areas of engineering, applied sciences, enterprise and finance, statistics, mathematics and research into operations, the exemplary exhibits in the real world show the random nature of the phenomena and the use of probabilistic techniques to model them accurately.

Probability theory, the theory of stochastic processes has been developed and it has been shown that practical problems can be applied successfully and empirical phenomena described. However, the theory is new and it is still necessary to discover the most appropriate mathematical techniques. It is therefore reasonable to hope that when more relevant mathematical problems are resolved, the usefulness of the theory will increase. These new problems, on the other hand, are also of interest to pure analysis beyond stochastic process theory. In the past, the interaction with physical theories has always benefited a great deal from pure mathematics, with many parts of the purst mathematics causing physical difficulties. We will see now that our theory leads to comprehensive differential equations of the kind never before studied: they contain a surprisingly wide variety of familiar and unknown functions as the simplest special cases. Another example of a general interest problem that we will touch upon shortly is that an empiric phenomenon often can be defined in several different ways , e.g. by a scheme of infinitely many ordinary differential equals. The fact is that the problem of the general interest we will deal with in short. This seems to indicate links that are still to be investigated.

As far as practical utility is concerned, it should be noted that it is absolutely not appropriate for a mathematical theory to include exact models of the phenomena observed. The constructive role of mathematical theories in applications is very often less important than the economy of thinking and experimentation, because mathematical arguments eliminate qualitatively reasonable working assumptions. Perhaps even more significantly, in the context of observations, a constant analysis of observation in the light of the theory and theory can thus become an essential guide, not only for better understanding but also for properly formulating scientific problems. Mathematical theory in geology, for example, we are presented with natural processes, some of which cover the earth's surface for millions of years. We note that some species are experiencing a period of prosperity and continuous growth and only die suddenly and unreasonably. Is it really necessary for every new observation to introduce new hypotheses, to assume cataclysms that work unilaterally against certain species, or to find other explications? The Volterra-Lotka theory of fighting for life indicates that circumstances that are apparent to the naïve observer, just like many geologic catastrophes, are bound to occur even under constant conditions. Even the simplest mathematical model of a stochastic process, combined with observations of age, geographical distribution and sizes, allows us to deduce valuable information on the influence on the developments of different factors like selection, mutations, and l while it is impossible to give a precise mathematical theory of evolution. This complements undecisive qualitative claims with a more persuasive quantitative analysis.

A stochastic process is called measurable, if $X: T \times \Omega \to S$ is measurable with respect to the product σ-algebra $B(T) \times A$. In the case of a real-valued process ($S = R$), one says X is continuous in probability if for any $t \in R$ the limit $X_{t+h} \to X_t$ takes place in probability for $h \to 0$. If the sample function $X_t(\omega)$ is a continuous function of t for almost all ω, then X_t is called a continuous stochastic process. If the sample function is a right continuous function in t for almost all $\omega \in \Omega$, X_t is called a right continuous stochastic process. Two stochastic process X_t and Y_t satisfying $P[X_t - Y_t = 0] = 1$ for all $t \in T$ are called modifications of each other or indistinguishable. This means that for almost all $\omega \in \Omega$, the sample functions coincide $X_t(\omega) = Y_t(\omega)$.

Let (Ω, A,P) be a probability space and let $T \subset R$ be time. A collection of random variables X_t, $t \in T$ with values in R is called a stochastic process. If X_t takes values in $S = R^d$, it is called a vector-valued stochastic process but one often abbreviates this by the name stochastic process too. If the time T can be a discrete subset of R, then X_t is called a discrete time stochastic process. If time is an interval, R^+ or R, it is called a stochastic process with continuous time. For any fixed $\omega \in \Omega$, one can regard $X_t(\omega)$ as a function of t. It is called a sample function of the stochastic process. In the case of a vector-valued process, it is a sample path, a curve in R^d.

The simplest examples

(1) The Poisson process.-Physicists, who sometimes term it "random events," often term it the Poisson distribution following Bateman, know this well. Here we take this example as a starting point for numerous generalizations and emphasize that the all-important distribution of Poisson is part of its privileges, rather than merely an approximation of binomial distribution.

Denote the likelihood of n events in a time interval of length t by Pn(t). We would like the P.(t + dt) to be compared. In one way, n 1 occurs at the interval (0, t + dt): either in n events (0, t) and in n events (t, t + dt) or in n + (0, t) or in one event (t, t + dt); or in n 1 event in (0, t) or in (t, t + dt) in n + dt. Writing down the corresponding probabilities we find

$$P_n(t + dt) = (1 - \eta dt)P_n(t) + \eta dt P_{n-1}(t) + o(dt), \qquad (1)$$

and similarly

$$P_o(t + dt) = (1 - \eta dt)P_o(t) + o(dt). \qquad (1')$$

Rearranging these equations and passing to the limit we find easily that our probabilities satisfy the system of differential equations

$$\begin{cases} P'_o(t) = -\eta P_o(t), \\ P'_n(t) = -\eta P_n(t) + \eta P_{n-1}(t), \qquad n \geqq 1. \end{cases} \qquad (2)$$

The initial conditions are obviously

$$P_o(0) = 1, \qquad P_n(0) = 0, \qquad n \geqq 1. \qquad (3)$$

Fortunately, in this case, the differential equations are of a recursive character and can be solved successively. The required solution is given by the familiar Poisson distribution

$$P_n(t) = e^{-\eta t}\frac{(\eta t)^n}{n!}, \qquad n \geqq 0; t \geqq 0. \qquad (4)$$

$$M(t) = \eta t, \qquad \sigma^2(t) = \eta t. \qquad (5)$$

It should be remembered that the relevant distribution of Poisson (4) is much broader than on the surface. In such cases, the t parameter is t for volume, area or length instead of time, and therefore describes many phenomena that play in space, rather than in a time. In this way, we can see that many of our assumptions, which resulted in differential equations (2), include the distribution of stars in space, material defects, raisins of a cake and misprints in a book. In these cases we will use the 'operative time' parameter t. We will see that many stochastic procedures are not necessarily perforated in time, but can be operative in any way, such as penetration depth, strength or so on.

$$\begin{cases} P'_o(t) = -\eta_o P_o(t), \\ P'_n(t) = -\eta_n P_n(t) + \eta_{n-1} P_{n-1}(t), \qquad 1 \leqq n \leqq N; \eta_N = 0. \end{cases} \quad (6)$$

Naturally, the initial conditions are the same as before. Systems (6) are also present in literature, and several writers have independently provided the explicit solution.If we suppose that no two among the vi are equal, this solution can be written in the form

$$P_n(t) = (-1)^n \eta_o \eta_1 \cdot \cdot \cdot \eta_{n-1} \times$$
$$\sum_{k=0}^{n} e^{-\eta_k t} \{ (\eta_k - \eta_o) \cdot \cdot \cdot (\eta_k - \eta_{k-1}) (\eta_k - \eta_{k+1}) \cdot \cdot \cdot (\eta_k - \eta_n) \}^{-1}. \quad (7)$$

$$\eta_n = n\eta, \quad (8)$$

and (6) becomes

$$P'_n(t) = -n\eta P_n(t) + (n-1)\eta P_{n-1}(t), \qquad n > 1, \quad (9)$$

usually with the natural initial condition

$$P_1(0) = 1, \qquad P_n(0) = 0, \qquad n > 1. \quad (10)$$

The solution follows either from (7) or by direct integration:

$$P_n(t) = e^{-\eta t}(1 - e^{-\eta t})^{n-1}. \quad (11)$$

In the mathematical theory of evolution that we alluded to in the introduction, Yule has used this kind of method (in an indirect manner). The population is made up of the species within a genus or of the specific animal or plant genera. It is due to mutations that each species or genus has a (constant) chance to cast a new form or genus at any time. This theory is used to analyze relationships between the generic period, the number of species composing, its geographic distribution, etc.

PROBABILITY THEORY

Probability theory is the main element of modern mathematics with the relationships between algebra, topology, analysis, geometry or dynamic structures in other mathematical fields. Theory begins with adding more structure to a set, as with all basic mathematical structures. In the same way as introduce algebraic operations, topology, or time evolution on a set, probability theory introduces a theoretical measure structure to which "count" is generalized in finite sets: in order to quantify a subset 's probability A as a consequence of each, one chooses a sub-set class A that one can expect to do. This leads to a −algebra A. This leads to This is a sequence of subsets of bars in which several operations such as unions, replacements or cross-sections may be carried out finely or counted. The A elements are referred to as events. When point ± denotes a "experiment" in "laboratory" a "case" A to A is a subset of "je," for which a probability P[A] to [0 , 1] can be assigned. If for instance P[A] = 1/3, the event is 1/3 probable. If P[A] = 1, it is almost certain that the event takes place. P must satisfy simple properties such as the union A = B of two disjointed events A, the probability measure B satisfies P[A − B] = P[A] + P[B] or the probability P[Ac] = 1 − P[A] of the addition Ac of an event A Chance. There are already some fascinating mathematics with a space of probability alone (A, A, P), for example, the combination complexity of calculating the likelihood of events to get "royal flush" in poker. If Â t is a subset of a euchlidean space like a plane, then we have problems with integration in calculus, P[A]= R A f(x, y) dxdy for a suitable none-negative function f. The probability space is simply part of the Euclidean space in many applications and the μ-algebre is the smallest space containing all open sets. The Borel μ-algebra is named. The Borel Ś-algebra on the real line is an important example.

Due to the space of a probability (fixed, A, P), random variables X can be defined. A random variable is a function X from a perspective to the actual line R which can be calculated such that the opposite in a measurable Boron set B in A is calculated. The definition is that if altern is an experiment, then X(alternatively) calculates a quantity found in the experiment. Measurability is technically consistent with continuity of the function f from the topological space (demons, o) to

the topological space (r, o). If f −1(U) − O for all open sets U − U is continuous, a function is constant. In the theory of probability, functions with capital letters like X , Y, are sometimes denoted If X−1 (B) − A for all Borels sets B − B − the random variable X can be measured. For Borel ̈-algebra any continuous function can be evaluated. Like in calculus, where continuity most of the time is not to be worried, in theory of probability also, measuring problems often don't have to be sweated about. In addition, notions such as ̈ algebras or measurability by mathematicians can be accused of driving ordinary people away from their realms. That's not the condition. With these constructions, severe problems are avoided. Mathematics is eternal: in thousands of years a once established outcome will be true. A theory that would prove both a theorem and its rejection is valuable: theoretically it would be possible to prove any other, true or false, results. So not only are these notions introduced to keep the theory "clean," but they are indispensable for the theory's "survival." We give some examples of "paradoxes" to show that a careful theory needs to be constructed. Returns to the theory of random variables: due to their fact that they are simply functions, one can add and multiply them by describing them (X+Y)(function)
=X(alternative)+Y(alternative)(Alternative))or(XY)(alternative)=X(alternative)Y(alternative).
The algebra L forms random variables. If it exists, E[X] denotes expectations of a random variable X. This is an actual number that shows the "mean" or "average" of observation X. It's the value that one would expect the experiment to calculate. If X = 1B is the random variable which holds 1 if iv is in case B and 0 if iv is not in case B, then X is only supposed to have B. The random constant variable X(dir) = an is predicted E[X] = a. All of these fundamental definitions and the linearity criterion of E[aX+bY] = aE [X] + bE[Y], describe the expectation of all random L variables: firstly, the expectation for finite sums called elementary random variables Pn i=1 ai1bi specifies, with general measurable functions approximating, the expectation. Extending the L-1 expectation of the whole algebra to a subset is part of the integration theory. While the calculus shows that one can live on the actual line with an integrated Riemann which defines the integral by the Riemann sums b a f(x) Dx = 1 n P I / n: The second is more important and the theory of probability is a major incentive to use it. It makes statements such as that the likelihood of the set of real numbers with normal decimal expansion is 0. The possibility of A is generally anticipated in X(x) = f(x) = 1A(x). The integral R 1 0 f(x) dx is not defined in calculus, since Riemann can provide 1 or 0 depending on how the

Riemann approximation is made. Probability theory enables the integration of Lebesgue by defining Rb a f(x) dx as a 1 n Pn i=1 f(xi) limit for n ~ , when xi is randomly randomly distributed in the same interval [a, b]. This definition of the Lebesgues Integral by Monte Carlo is based on the law of large numbers as the Riemann Integral, the limit 1 n P xj = j / n alternatives[a, b] f(xj) for n x lifes. This is the Riemann Integral.

SOME APPLICATIONS OF PROBABILITY THEORY

Probability theory is a central topic in mathematics. In other areas such as computer science, ergodic theory, dynamic system, cryptology, game theory, analysis, partial difference equation, physics, economics, statistical mechanics and also number theory. We give you some problems and topics that can be treated by probabilistic methods as a motivation.

1) Random walks: Suppose you're going through a cloth. You pick a random path from each vertex. How likely are you to return to your point of departure? The theorem of Polya states that a random walker almost certainly arbitrarily returns the origin in two dimensions while the walker with probability 1 returns only a few times and escapes forever in 3 dimensions.

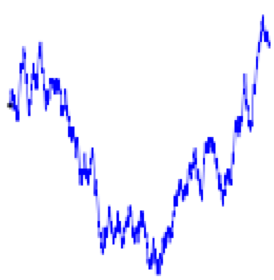

Figure. *A random walk in one dimensions displayed as a graph* (t, B_t).

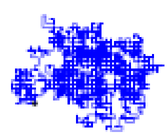

Figure. *A piece of a random walk in two dimensions.*

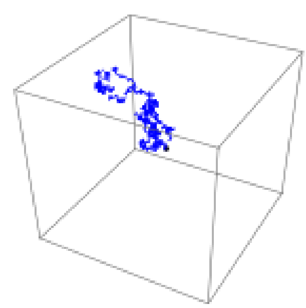

Figure. *A piece of a random walk in three dimensions.*

2) Percolation problems (model of a porous medium, statistical mechanics and critical phenomena). The probability p is associated to each bond of the rectangular mesh in the plane, and 1 – p is disconnected. If there is a trail from x to y, two lattice points x , y in the lattice are in the same cluster. One says that "percolation takes place" if there is a positive chance of an endless cluster. The critical probability pc, the lowest of all ps, for which percolation is present, is one question. The problem can be extended to situations where the probabilities of switching are not mutually exclusive. Some random variables are important near critical probability pc, including the size of the largest cluster.

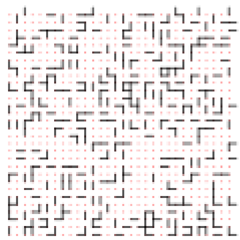

Figure. *Bond percolation with p=0.2.*

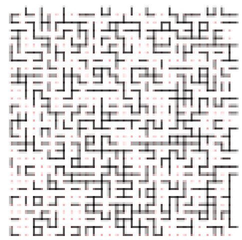

Figure. *Bond percolation with p=0.4.*

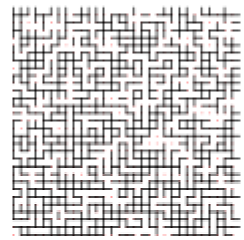

Figure. *Bond percolation with p=0.6.*

A variant of bond percolation is site percolation where the nodes of the lattice are switched on with probability p.

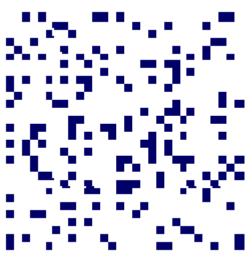

Figure. *Site percolation with p=0.2.*

Figure. *Site percolation with p=0.4.*

Figure. *Site percolation with p=0.6.*

Generalized percolation problems are obtained, when the independence of the individual nodes is relaxed. A class of such dependent percolation problems can be obtained by choosing two irrational numbers α, β like $\alpha = \sqrt{2} - 1$ and $\beta = \sqrt{3} - 1$ and switching the node (n, m) on if $(n\alpha + m\beta) \bmod 1 \in [0, p)$. The probability of switching a node on is again p, but the random variablesare no more independent.

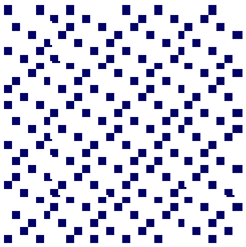

Figure. *Dependent site percolation with p=0.2.*

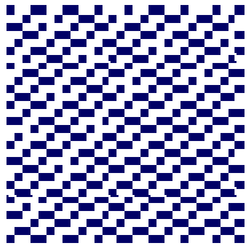

Figure. *Dependent site percolation with p=0.4.*

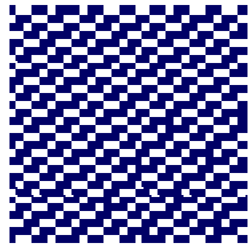

Figure. *Dependent site percolation with p=0.6.*

3) Random Schr¨odinger operators. (Quantum mechanics, functional analysis, disordered systems, solid state physics) Consider the linear map Lu(n) = P |m−n|=1 u(n) + V (n)u(n) on the space of sequences u = (. . . , u−2, u−1, u0, u1, u2, . . .). We assume that V (n) takes random values in {0, 1}. The function V is called the potential. The problem is to determine the spectrum or spectral type of the infinite matrix L on the Hilbert space l 2 of all sequences u with finite ||u||2 2 = P∞ n=−∞ u 2 n. The operator L is the Hamiltonian of an electron in a one-dimensional disordered crystal. The spectral properties of L have a relation with the conductivity properties of the crystal. Of special interest is the situation, where the values V (n) are all independent random variables. It turns out that if V (n) are IID random variables with a continuous distribution, there are many eigenvalues for the infinite dimensional matrix L - at least with probability 1. This phenomenon is called localization.

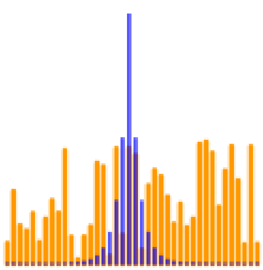

Figure. *A wave $\psi(t) = e^{iLt}\psi(0)$ evolving in a random potential at $t = 0$. Shown are both the potential V_n and the wave $\psi(0)$.*

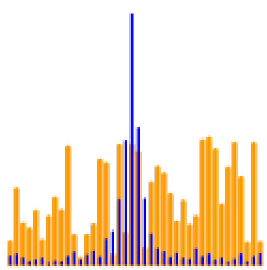

Figure. *A wave $\psi(t) = e^{iLt}\psi(0)$ evolving in a random potential at $t = 1$. Shown are both the potential V_n and the wave $\psi(1)$.*

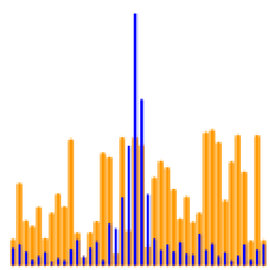

Figure. *A wave $\psi(t) = e^{iLt}\psi(0)$ evolving in a random potential at $t = 2$. Shown are both the potential V_n and the wave $\psi(2)$.*

4) Classical dynamical systems (celestial mechanics, fluid dynamics, mechanics, population models) Studies of dynamic deterministic systems such as the logistic map x 7 to 4x (1 − x) on [0

, 1] interval or the 3 celestial mechanical body problem have shown that such systems or sub-assemblies can act as a random mechanism. Many effects can be described through ergodic theory, which can be seen as a probability theory brother. Several findings in the theory of probability generalizing ergodic theory to more general configuration. An example is the ergodic theorem of Birkhoff that generalizes the law of large numbers.

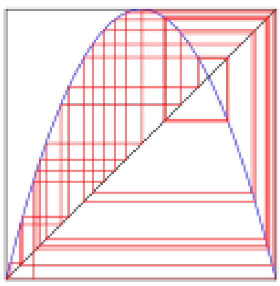

Figure. *Iterating the logistic map*

$$T(x) = 4x(1-x)$$

on $[0,1]$ *produces independent random variables. The invariant measure* P *is continuous.*

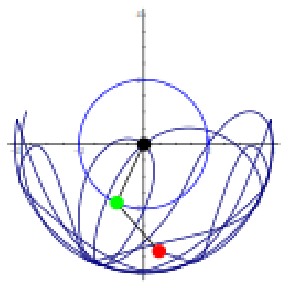

Figure. *The simple mechanical system of a double pendulum exhibits complicated dynamics. The differential equation defines a measure preserving flow* T_t *on a probability space.*

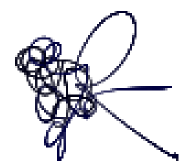

Figure. *A short time evolution of the Newtonian three body problem. There are energies and subsets of the energy surface which are invariant and on which there is an invariant probability measure.*

Given a dynamical system given by a map T or a flow Tt on a subset Ω of some Euclidean space, one obtains for every invariant probability measure P a probability space (Ω, A,P). An observed quantity like a coordinate of an individual particle is a random variable X and defines a stochastic process Xn(ω) = X(T nω). For many dynamical systems including also some 3 body problems, there are invariant measures and observables X for which Xn are IID random variables. Probability theory is therefore intrinsically relevant also in classical dynamical systems.

5) Cryptology. (Computer science, coding theory, data encryption) The coding theory concerns the coding mathematics or the nature of error correction codes. There are important applications in both aspects of coding theory. A good code can repair data loss due to poor channels and encrypt information. In many respects, the coding theory is based on discrete mathematics, the theory of the number, the algebra and algebraic geometry. We illustrate this with a public key encryption algorithm, the safety of which is based upon the fact that it isn't easy to fill a large integer N = pq into its primary p factors. The N can be public, but only the individual with factors p knows q can read the message. Suppose we want to split the code and find p and q factors.

The simplest way is to test and error for the factors. But if N has 50 digits, it is already not practical. To find the p factor, we would need to look for 1025 numbers. This is 100 million times a second over 15 billion years. That corresponds 100 million times per second. More information is being given and one of them should now be illustrated: assume that we want to find N = 1111111111111111111111111111111111111111111111111. This method is the following: start by integrating the T map (x) = x 2 + c mod N on {0 , 1.,,,,, N − 1}. If x0 = an is assumed, x1 = T (a), x2 = T(a) is assumed. How many numbers do we need to generate in order to be random, before two of them are one of the prime factors p? The response to the birthday paradox is surprisingly small: in a group of 23 students, two are over 1/2 years old; the chance for the case that we have no match is 1(363/365)• •(343/365)= 0.492703. •(343/365) . the probability is 1 − 0.492703 = 0.507292 for a birthday match. This exceeds 1/2. If we apply this way of thinking to the sequence of numbers xi generated by PRGT, we expect a 1/2 chance of finding a matching modulo p in oscillations. Since p is the same as that of the module p, then we must try N1/4 to obtain a factor: if xn and xmare the same p module, then the p factor p is produced. A first factor of p = 35121409 can be found in the above example of a 46-digit number N. This factor is found in the Pollard algorithm in Ã lp = 5926 steps with probability of 1/2. This is an estimate of the magnitude only. If you start with a = 17 and get a = 3, you get x27720 = x13860 at the above N. It is very quickly found. When we actually choose random numbers, this probabilistic claim will have a detailed probabilistic calculation. Obviously, these numbers are generated in a deterministic way by the algorithm and not really random. A pseudo random number generator is called the generator. It generates random numbers in the sense that various statistical tests can not differentiate between them and true random numbers. In addition, many random numbers generators integrated into computation and programming systems are pseudo-random numbers generators.

6) Numerical methods. (Integration, Monte Carlo experiments, algorithms) In applied situations, it is often very difficult to find integrals directly. This happens for example in statistical mechanics or quantum electrodynamics, where one wants to find integrals in spaces with a large number of dimensions. One can nevertheless compute numerical values using Monte Carlo Methods with a manageable amount of effort. Limit theorems assure that these numerical values are reasonable. Let us illustrate this with a very simple but famous example, the Buffon needle problem.

CONCLUSION

Stochastic processes are generally treated in the mathematical literature formally and in general, not clearly indicating the practical meaning and applicableness. On the contrary, practical problems leading to stochastic processes are usually dealt with using special methods and under different disguise, thus making no apparent connection to the general theory. So the most simple and intuitive examples of sthetic processes discussed in the literature are to be explained. It seems advisable. They do not need new mathematical instruments since they all lead to systems of ordinary differential equations that are very simple, although infinite. We will then move on to general theories, but the most general type of stochastic processes such as occur in time-series analyses will not be taken into account in this paper. Instead, we shall limit considerations in the same way that the current state determines the system 's future evolution in classical mechanisms to what are now generally known as Markov processes , i.e. to processes where all future relations between probability are fully established by the current state. In addition, the so-called discontinuous type of Markov processes where changes occur in jumps will focus our attention: the system will stay intact for a while and subsequently suddenly change into a new State. These processes have found important and extensive applications in automatic telephone theory and in insurance risk theory.Theory of Probability," the theory of stochastic processes has been developed and it has been shown that it can successfully be applied to practical problems and used to describe empirical phenomena. However, the theory is new and the most appropriate mathematical techniques have yet to be discovered. It is therefore reasonable to expect that the usefulness of the theory will increase when more pertinent mathematical problems are solved. On the other hand, these new problems are of interest also in pure analysis beyond the theory of stochastic processes.

REFERENCES

1. S. C. Pryor, M. Nielsen, R. J. Barthelmie, and J. Mann, “Can satellite sampling of off shore wind speeds realistically represent wind speed distributions? Part II: Quantifying uncertainties associated with distribution fitting methods,” J. Appl. Meteorol. Climatol.43, 739–750 (2004).

2. H. T. Yura, “LADAR detection statistics in the presence of pointing errors,” Appl. Opt. 33, 6482–6498 (1994).

3. L. C. Andrews, R. L. Phillips, and C. Y. Hopen, Laser Beam Scintillation with Applications (SPIE, 2001), Chap. 2.

4. V. I. Tatarskii, The Effects of the Turbulent Atmosphere on Wave Propagation (Israel Program for Scientific Translations, 1971).

5. The CDF of the Rice–Nakagami distribution can be expressed in terms of a Marcum Q function, which are tabulated, but not supported, to the best of our knowledge, by any commercial commuter programs such as Mathematica and MATLAB.

6. J. W. Goodman, Speckle Phenomena in Optics: Theory and Applications, (Roberts, 2006), Section 3.2.2.

7. D. V. Kiesewetter, “Numerical simulation of a speckle pattern formed by radiation of optical vortices in a multimode optical fibre,” Quantum Electron. 38, 172–180 (2008).

8. A. Garcia-Martin, R. Gomez-Medina, J. J. Saenz, and M. Nieto-Vesperinas, “Finite-size effects in the spatial distribution of the intensity reflected from disordered media,” Phys. Rev. B 62, 9386–9389 (2000).

9. J. W. Goodman, “Speckle with a finite number of steps,” Appl. Opt. 47, A111–A118 (2008).

10. H. Satoh, K. Sekiya, T. Kawakami, Y. Kuratomi, B. Katagiri, Y. Suzuki, and T. Uchida, “On the effect of small moving diffusers to the speckle reduction in laser projection displays,” in IDW ’09—Proceedings of the 16th International Display Workshops (2009), pp. 1361–1364.

11. P. Lehmann, A. Schone, and J. Peters, “Non-Gaussian far field fluctuations in laser light scattered by a random phase screen,” Optik 95, 63–72 (1993).

12. J. W. Goodman, Speckle Phenomena in Optics: Theory and Applications (Roberts, 2006), Section 3.6.

13. M. Shinozuka and G. Deodatis, “Simulation of stochastic processes by spectral representation,” Appl. Mech. Rev. 44, 191–204 (1991).

14. J. W. Goodman, Speckle Phenomena in Optics: Theory and Applications (Roberts, 2006).

