SOME RESULTS ON HAUSDROFF TOPLOGICAL SPACE

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ABSTRACT: The Purpose of this paper is to establish a few results on Hausdroff Topological space, rotuned, Sublevel, inf-compact.

1.1 **INTRODUCTION** : In order to prepare this paper efforts has been made in studying Hausdroff Topological space in detail. Attentions has also been made to study the notion of convex function and inf. Compact.

1.2 **DEFINATION** : For definitions we refer to Jha (1) and Simmons (1).

However to serve as ready reference we give below same of the definitions.

CONVEX FUNCTION : Let X be Topological vector space and R be the set of all real numbers. A mapping $f : X \to R$ is called a convex function if for all

$x_1, x_2 \in X$ and $p \in R$, 0<p<1 we have

$f(px_1 + (1-p)x_2) \leq pf(x_1) + (1-p)f(x_2)$.

HAUSDROFF TOPOLOGICAL SPACE : A topological space (X,T) is called a Hausdroff Topological space (or Simply Housdroff space) or separated space if for each pair x and y of distinct points of X, There exists a pair G,H of open sets such that $x \in G, y \in H$ and $G \cap H = \phi$

The Topology of a Hausdroff space is called a Hausdroff topology.

Inf- **COMPACT AT A POINT** :- A set K is said to be inf-compact at a point $x \in X$, if each minimizing net K_α in K, $\{f(x, K_\alpha) \to f(x, K)\}$ has a convergent subnet converging in K.

Inf- **COMPACT:** The set K is said to be inf-compact if it is Inf- compact at each point $x \in X$.

ROTUND: A real valued convex function f defined on a Hausdroff linear topological space X is said to be rotund if the sub-level sets $S_\alpha(\alpha \in R)$ of f are rotund, that is $f(x) = f(y) = \alpha$ implies $f\{\beta x + (1-\beta)y\} < \alpha \; for \; 0 < \beta < 1$.

1.3 In this section we establish we establish a few results using the definitions given in section 2.

Theorem (1.3,I)

Let us assume that:

(1) K be a non-empty subset of a Hausdroff Topological space X

(2) F be a real valued function on XxX.

(3) K is inf-Compact at a point $x \in X$ and $x \to f(y_0, x)$ be lower semi-continuous.

Then $p(x,K)$ is a non-empty compact subset of K

Proof : By the definition of $\mathrm{f}(x,K)$, we can extract a net $\{K_\alpha\}$ in K

Such that $\lim_\alpha f(x, \mathrm{K}_\alpha) = \mathrm{f}\{x, \mathrm{K}\}$.

Since by assumption K is inf-compact at a point x.

Hence $\{K_\alpha\}$ has a convergent subset say $\{K_\beta\}$ converging in K.

Again let $K_\beta \to K_\alpha$ in K.

Then $f(x, K_0) \leq \lim_\alpha f(x, K_\alpha) = f(x, K)$

Thus $f(x, K_0) = f(x, K)$ which gives $K_0 \in p(x, K)$

Thus the compactness of $p(x,K)$ follows trivially from the definition of inf-compactness of K at x.

Theorem (1.3,II):

Let (i) K be a non-empty subset of a Hausdroff topological space X

(ii) f be a real valued function on X x X.

(iii) K be an inf-compact at a point y_0 in X.

(iv) The mapping $x \to f(y_0, x)$ be lower semi continuous

(v) $T : K \xrightarrow{\text{into}} K$ such that

(a) $f(y_0, T_x) \leq f(y_0, x), x \in K$

(b) for each compact subset F of K which contains more than one element and is mapped into itself by T, There is an element x_0 in F such that $f(y_0, \mathrm{F}) < f(y_0, x_0)$

Then T has a fixed point y in K which is f-nearest to y_0 in K.

Proof: By [Theorem (1.3,I) $P(y_0, K)$ is a non-empty compact subset of K, Also by [hypothesis (v) (a)] it is mapped into itself by T.

Let F= The family of all non-empty closed subsets of $P(y_0, K)$ each of which is mapped in to itself under the mapping T.

Now ordering f by set inclusion, by compactness of $P(y_0, K)$ and Zorn's lemma, we obtain a minimal element F of $P(y_0, K)$.

Let F contains more than one element

Also, for $x \in P(y_0, F), f(y_0, Tx) \leq f(y_0, x) = f(y_0, F)$ which implies that

$$Tx \in P(y_0, F)$$

Thus $P(y_0, F) \in F$

But by [hypothesis (v) (b)] $P(y_0, F)$ is a proper subset of F but it contradicts minimality of F

Hence F contains a unique element y such that $Ty = y$.

Also F is a subset of $P(y_0, F)$, we get that

$$f(y_0, \mathrm{y}) = f(y_0, \mathrm{K})$$

Thus the theorem is established.

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