An Integrated Vendor-Buyer Inventory Model for Deteriorating Items with Two Part Trade Credit When Demand is Time Dependent

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Abstract

In this article, we develop an integrated vendor-buyer inventory model with the assumption that market demand is linear with respect to time and vendor adopts a trade credit policy. Items in the inventory are subject to deteriorate with constant rate. The buyer has an option to choose between discount in unit price and delay in settling the account against the purchase made offered by the vendor. This type of trade credit is termed as 'two-part trade credit'. That is, if the buyer pays within M1, the buyer receives a cash discount; otherwise the full payment must be paid before M2, where M2 >M1 ≥ 0. Here mathematical model have been derived for obtaining the optimal cycle time for item so that joint total profit of the supply chain is maximised. Furthermore, numerical example is given to illustrate the results and managerial insights are drawn.

Keywords: Integrated inventory model, Cash discount, trade credit, linear demand, deteriorating items.

1. Introduction

While developing a mathematical model in inventory control, it is assumed that payment will be made to the supplier for the goods immediately after receiving the consignment. However it is found that trade credit is an important source of financing for intermediate purchasers of goods and services. Here supplier allows a certain fixed period to settle the account. Before the end of trade credit period, the retailer can sell the goods and accumulate revenue and earn interest. During this fixed period no interest is charged by the supplier, but beyond this period interest is charged by the supplier under the agreed terms and conditions. Goyal (1985) was the first to establish an economic order quantity model with a constant demand rate under the condition of permissible delay in payments. Ventura (1985) then suggested to extend the model by considering a customer trade credit offered by the retailer to its customers. In the same year Dave (1985) corrected Goyal's model by assuming the fact that the selling price is necessarily higher than its purchase price. Aggarwal and Jaggi (1995) then extended Goyal's model for deteriorating items. Jamal et al. (1997) further generalised the model to allow for shortages and deterioration. Hwang and Shinn (1997) developed the optimal pricing and lot sizing for the retailer under the condition of permissible delay in payments. After that, numerous studies dealing with the trade credit problem have been presented. For example, Chang and Dye (2001), Chang et al. (2003), Goswami et al. (2010), Jaggi et al. (2010),Tsao (2011), Sarkar (2012), Teng et al. (2012), Cheng et al. (2012), Zhong and Zhou (2012), Chern et al. (2013), Huang et al. (2013), Moussawi-Haidar et al. (2014), Chern et al. (2014), Li et al. (2014), Majumder et al. (2015), Thangam (2015) and so on. One can refer review by Kawale and Sanas (2017) on trade credit and inventory policy.

Ho *et al.* (2008) discussed ''two-part'' strategy: cash discount and delayed payment. That is, if the buyer pays within M1, the buyer receives a cash discount; otherwise, the full purchasing price must be paid before M2, where M2 >M1 ≥ 0. To accelerate cash inflow and reduce the risk of a cash crisis and bad debt, the supplier may provide a cash discount to encourage the buyer to pay for goods quickly. Thus in Ho et al. (2008), the supplier offered a ''two-part'' trade credit to the buyer to balance the trade off between delayed payment and cash discount. For example, under a contract, the supplier agrees to a 2% discount deducted from the buyer's purchasing price if payment is made within 10 days. Otherwise, full payment is required within 30 days after the delivery. This credit term in financial management is denoted as ''2/10 net 30''. There are more papers related to this trade credit policy such as Chang (2002) , Huang and Chung (2003), Ouyang et al. (2002), Huang and Hsu (2007), Jain et al. (2008), Shah and Shukla (2011), Shah et al. (2013), Kumar et al. (2011).

Lee et al. (1997) argued that without coordinated inventory management in the supply chain may result in excessive inventory investment, revenue reduction and delays in response to customer satisfaction. Therefore, the joint discussion is more beneficial as compared to the individual decision. Goyal (1976) first developed an integrated inventory model for a single supplier – single customer problem. Banerjee (1986) extended Goyal's (1976) model under assumption of a lot – for – lot production for the vendor. Later Goyal (1988), Lu (1995), Bhatnagar et al. (1993), Goyal (1995), Viswanathan (1998), Hill (1997, 1999), Kim and Ha (2003), Kelle et al. (2003), Li and Liu (2006) developed an integrated inventory model. These studies on integrated inventory problems did not take the effect of trade credit on the optimal policy between the supplier and buyer into account. Abad and Jaggi (2003) first offered a supplier–buyer integrated model following a lot-for-lot shipment policy under a permissible delay in payment.

In this paper, the objective is to analyze an integrated inventory system for deteriorating items for linear time dependent demand. The units in inventory are subject to deterioration at a constant rate. The vendor offers a choice of cash discount in unit purchase price if payment is settled earlier; otherwise, the buyer has to make the full payment by the allowable credit period. The joint total profit per unit time is maximized with respect to the cycle time. A numerical example is given to validate the developed problem. Sensitivity analysis is carried out and managerial issues are discussed.

2. Notations

The following notations are used in the proposed article:

S_v: Vendor's set up cost per set up.

S_b: Buyer's ordering cost per order.

C_v: Production cost per unit.

C_b: Buyer's purchase cost per unit.

C_c: The unit retail price to customers where $C_c > C_b > C_v$.

I_v: Vendor's inventory holding cost rate per unit per annum, excluding interest charges.

I_b: Buyer's inventory holding cost rate per unit per annum, excluding interest charges.

I_{v0}:Vendor's opportunity cost/$/unit time.

I_{b0}: Buyer's opportunity cost/$/unit time.

I_{be}: Buyer's interest earned/$/unit time.

ϱ: Capacity utilisation which is ratio of demand to the production rate, ϱ <1 and known constant.

M_1 : Period of cash discount

M_2: Allowable credit period for the buyer offered by the vendor. ($M_2 > M_1$)

Q: Buyer's order quantity.

T: cycle time (decision variable).

n : Number of shipments from vendor to the buyer.

ɵ: constant rate of deterioration.

ʎ: cash discount rate

fvc: Vendor's cash flexibility rate

TVP: Vendor's total profit per unit time.

TBP: Buyer's total profit per unit time.

π : TVP + TBP Joint total profit per unit time.

3. Assumptions

In addition, the following assumptions are made in derivation of the model:

- The supply chain under consideration comprise of single vendor and single buyer for a single product.
- Shortages are not allowed.
- The demand rate considered is time dependent, increasing demand rate. The constant part of linear demand pattern changes with each cycle.
- Replenishment rate is instantaneous for buyer
- The units in inventory are subject to deteriorate at a constant rate of θ, $0 < \theta < 1$. The deteriorated units can neither be repaired nor replaced during the cycle time.
- Finite production rate.
- Vendor produces the nQ items and then fulfils the buyer's demand, so at the beginning of production item, there is small possibility of deterioration in general. Moreover vendor is a big merchant who can have capacity to prevent deterioration. So in this model, deterioration cost is considered for buyer only at the rate θ is assumed to be constant.
- The vendor offers a discount λ ($0 < \lambda < 1$) in the purchase price if the buyer pays by time M1; otherwise full account is to be settled within allowable credit period M2, where M2 >M1 $\geq$ 0. The offer of discount in unit purchase price from the vendor will increase cash in-flow, thereby reducing the risk of cash flow shortage.
- By offering a trade credit to the buyer, the vendor receives cash at a later date and hence incurs an opportunity cost during the delivery and payment of the product. On the buyer's end, the buyer can generate revenue by selling the items and earning interest by depositing it in an interest bearing account during this permissible delay period. At the end of this period, the vendor charges to the buyer on the unsold stock.
- During the time [M1,M2], a cash flexibility rate fvc is used to quantize the favor of early cash income for the vendor.

4. Mathematical Model

The inventory level at any instant of time *t* is governed by the differential equation

$$\frac{dI(t)}{dt} + \theta\, I(t) = -(a + bt) \qquad 0 \leq t \leq T$$

With boundary condition I(0) =Q and I(T)=0 , we get:

$$Q = \left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)(e^{\theta T}-1) + \frac{bT}{\theta}e^{\theta T}, \qquad 0 \leq t \leq T.$$

$$I(t) = \left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)(e^{\theta(T-t)}-1) + \frac{b}{\theta}(Te^{\theta(T-t)} - t), \qquad 0 \leq t \leq T.$$

4.1 Net profit function for vendor consists of following elements:-

For each unit of item, the vendor charges $((1 - k_j\Lambda)C_b)$ if the buyer pays at time M_j, j=1,2, k_1=1 and k_2=0.

1. Sales revenue: the total sales revenue per unit time is $\left((1 - k_j\Lambda)C_b - C_V\right)\frac{Q}{T}$.

$$= \frac{\left((1 - k_j\Lambda)C_b - C_v\right)}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)(e^{\theta T} - 1) + \frac{bTe^{\theta T}}{\theta}\right\}$$

2. Set-up cost : nQ units are manufactured in one production run by the vendor. Therefore the setup cost per unit time is $\frac{S_v}{nT}$

3. Holding cost : using method given by Joglekar (1988),vendor's average inventory per unit time

$$\frac{C_v(I_v + I_{v0})}{T}[(n-1)(1-\varrho) + \varrho]\left\{\left(\frac{-a}{\theta^2} + \frac{b}{\theta^3}\right)(1 + \theta T - e^{\theta T}) - \frac{b}{\theta^2}\left(T - Te^{\theta T} + \frac{\theta T^2}{2}\right)\right\}$$

4. Opportunity cost : opportunity cost per unit time because of offering permissible delay period is $\frac{(1-k_j\Lambda)C_b I_{v0} M_j Q}{T}$

$$= \frac{(1 - k_j\Lambda)C_b I_{v0} M_j}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)(e^{\theta T} - 1) + \frac{bTe^{\theta T}}{\theta}\right\}$$

However, if the buyer pays at M_1 –time, during M_2 - M_1 the vendor can use the revenue $((1-ʎ)C_b)$ to avoid a cash flow crisis. The advantage gain per unit time from early payment at a cash flexibility rate fvc is

$$\frac{k_j(1-ʎ)C_b fvc\ (M2 - M1)Q}{T}$$

$$=\frac{k_j(1-ʎ)C_b fvc\ (M2 - M1)}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}\right\}$$

Hence the total profit per unit time for vendor is = Sales revenue – Set up cost – Holding cost – Opportunity cost + Advantage gain

$$TVP_j = \frac{((1-k_jʎ)C_b-C_v)}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}\right\}-\frac{S_v}{nT}-\frac{C_v(I_v+I_{v0})}{T}[(n-1)(1-\varrho)+\varrho]\left\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}\right)(1+\theta T-e^{\theta T})-\frac{b}{\theta^2}\left(T-Te^{\theta T}+\frac{\theta T^2}{2}\right)\right\}-\frac{(1-k_jʎ)C_bI_{v0}M_j}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}\right\}+\frac{k_j(1-ʎ)C_b fvc\ (M2 - M1)}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}\right\} \quad \text{---------------------(1)}$$

$j = 1,2;\quad k_1=1,\ k_2 = 0$

4.2 Net profit function for the buyer consists of following elements:-

1. Sales revenue: The total sales revenue per unit time is $\frac{(C_c-(1-k_jʎ)C_b)Q}{T}$

$$=\frac{(C_c-(1-k_jʎ)C_b)}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}\right\}$$

2. Ordering cost : Ordering cost per unit time is $\frac{S_b}{T}$

3. Holding cost: The buyer's holding cost (excluding interest charges) per unit time is $\frac{(1-k_jʎ)C_bI_b}{T}\left\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}\right)(1+\theta T-e^{\theta T})-\frac{b}{\theta^2}\left(T-Te^{\theta T}+\frac{\theta T^2}{2}\right)\right\}$

4. Deteriorating cost : Deteriorating cost per unit time is $\frac{(1-k_jʎ)C_b}{T}\left[Q-\int_0^T(a+bt)dt\right]$

$$=\frac{(1-k_jʎ)C_b}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}-aT-\frac{bT^2}{2}\right\}$$

Based on the length of the payment time, two cases arise namely $M_j < T$ and $M_j \geq T$; j = 1,2

Case 1] When $M_j < T$; j = 1,2.

5. Interest earned per unit time during the period [0, M_j] is $\frac{I_{be}C_c}{T}\int_0^{M_j}(a+bt)tdt$

$$= \frac{I_{be}C_c}{T}\left[\frac{aM_j^{\,2}}{2} + \frac{bM_j^{\,3}}{3}\right]$$

6. Interest payable per unit time during time span [M_j, T] is $\frac{(1-k_j\lambda)C_bI_{b0}}{T}\int_{M_j}^{T} I(t)dt$

$$= \frac{(1-k_j\lambda)C_bI_{b0}}{T}\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}\right)\left(1+\theta\left(T-M_j\right)-e^{\theta\left(T-M_j\right)}\right)-\frac{b}{\theta^2}(T-Te^{\theta\left(T-M_j\right)}+\theta(\frac{T^2}{2}-\frac{M_j^{\,2}}{2})\}$$

Therefore profit of the buyer in this case can be expressed as :-

TBP_{j1}= Sales revenue – Ordering cost – Inventory carrying cost – Deteriorating cost + Interest earned – Interest paid.

$$TBP_{j1}= \frac{(C_c-(1-k_j\lambda)C_b)}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}\right\} - \frac{S_b}{T} - \frac{(1-k_j\lambda)C_bI_b}{T}\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}\right)(1+\theta T - e^{\theta T}) - \frac{b}{\theta^2}(T-Te^{\theta T}+\frac{\theta T^2}{2})\} - \frac{(1-k_j\lambda)C_b}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}-aT-\frac{bT^2}{2}\right\} + \frac{I_{be}C_c}{T}\left[\frac{aM_j^{\,2}}{2}+\frac{bM_j^{\,3}}{3}\right] - \frac{(1-k_j\lambda)C_bI_{b0}}{T}\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}\right)\left(1+\theta\left(T-M_j\right)-e^{\theta\left(T-M_j\right)}\right)-\frac{b}{\theta^2}(T - Te^{\theta\left(T-M_j\right)}+\theta(\frac{T^2}{2}-\frac{M_j^{\,2}}{2})\} \quad \text{-------------------------------(2)}$$

j= 1,2

Case2] When $M_j \geq T$; j = 1,2.

The first 4 components of the profit function remain same. The sixth cost component does not exist for $M_j \geq T$. The interest earned per unit time during time span [0, M_j] is

$$\frac{(I_{be}\ C_c)}{T}\{\int_0^T (a+bt)tdt + Q(M_j - T)\}$$

$= \frac{I_{be}C_c}{T}\left[\frac{aT^2}{2} + \frac{bT^3}{3}\right] + \frac{I_{be}C_c}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)(e^{\theta T} - 1) + \frac{bTe^{\theta T}}{\theta}\right\}(M_j\text{-}T)$

In this case profit for the buyer is given by

TBP_{j2} = Sales revenue – Ordering cost – Inventory carrying cost – Deteriorating cost + Interest earned.

$TBP_{j2} = \frac{(C_c-(1-k_j\lambda)C_b)}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)(e^{\theta T} - 1) + \frac{bTe^{\theta T}}{\theta}\right\} - \frac{S_b}{T} - \frac{(1-k_j\lambda)C_bI_b}{T}\{\left(\frac{-a}{\theta^2} + \frac{b}{\theta^3}\right)(1 + \theta T - e^{\theta T}) - \frac{b}{\theta^2}(T - Te^{\theta T} + \frac{\theta T^2}{2})\}\text{-}\frac{(1-k_j\lambda)C_b}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)(e^{\theta T} - 1) + \frac{bTe^{\theta T}}{\theta} - aT - \frac{bT^2}{2}\right\} + \frac{I_{be}C_c}{T}\left[\frac{aT^2}{2} + \frac{bT^3}{3}\right] + \frac{I_{be}C_c}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)(e^{\theta T} - 1) + \frac{bTe^{\theta T}}{\theta}\right\}(M_j\text{-}T)$ -------------------------(3)

j= 1,2

4.3 Joint total profit per unit time

In integrated system, the vendor and the buyer to take joint decision which maximizes the profit of the supply chain, the joint total profit per unit time for integrated system is

$\Pi_j = \quad \Pi_{j1} = TVP_j + TBP_{j1} \qquad M_j < T$

$\qquad \Pi_{j2} = TVP_j + TBP_{j2} \qquad M_j \geq T; j = 1,2.$

Considering $e^{\theta T} = 1 + \theta T + \frac{\theta^2T^2}{2}$

$TVP_j = ((1 - k_j\lambda)C_b - C_v - (1 - k_j\lambda)C_bI_{v0}M_j + k_j(1 - \lambda)C_bfvc\,(M2 - M1))(a + \frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}) - \frac{S_v}{nT} - C_v(I_v + I_{v0})[(n - 1)(1 - \varrho) + \varrho](\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta})$ ----------------(4)

$TBP_{j1} = (C_c - (1 - k_j\lambda)C_b)(a + \frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}) - \frac{S_b}{T} - (1 - k_j\lambda)c_bI_b(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}) - (1 - k_j\lambda)C_b\left(\frac{a\theta T}{2} + \frac{b\theta T}{2}\right) + \frac{I_{be}C_c}{T}\left[\frac{aM_j^2}{2} + \frac{bM_j^3}{3}\right] - (1 - k_j\lambda)C_bI_{b0}\left\{a\frac{(T-M_j)^2}{2T} - b\frac{(T-M_j)^2}{2\theta T} - b\left(-\frac{(T-M_j)}{\theta} - \frac{(T-M_j)^2}{2} + \frac{T}{2\theta} - \frac{M_j^2}{2\theta T}\right)\right\}$ -------------------(5)

$$TBP_{j2} = \left(C_c - (1-k_j\Lambda)C_b\right)(a+\frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}) - \frac{S_b}{T} - (1-k_j\Lambda)c_b I_b(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}) - (1-k_j\Lambda)C_b\left(\frac{a\theta T}{2} + \frac{b\theta T}{2}\right) + \frac{I_{be}C_c}{T}\left[\frac{aT^2}{2} + \frac{bT^3}{3}\right] + I_{be}C_c(M_j - T)(a+\frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2})$$

------------------(6)

$$\Pi_{j1} = ((1-k_j\Lambda)C_b - C_v - (1-k_j\Lambda)C_b I_{v0}M_j + k_j(1-\Lambda)C_b fvc\,(M2 - M1))(a+\frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}) - \frac{S_v}{nT} - C_v(I_v + I_{v0})[(n-1)(1-\varrho)+\varrho]\left(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}\right) + \left(C_c - (1-k_j\Lambda)C_b\right)(a+\frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}) - \frac{S_b}{T} - (1-k_j\Lambda)c_b I_b(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}) - (1-k_j\Lambda)C_b\left(\frac{a\theta T}{2} + \frac{b\theta T}{2}\right) + \frac{I_{be}C_c}{T}\left[\frac{aM_j^2}{2} + \frac{bM_j^3}{3}\right] - (1-k_j\Lambda)C_b I_{b0}\left\{a\frac{(T-M_j)^2}{2T} - b\frac{(T-M_j)^2}{2\theta T} - b\left(-\frac{(T-M_j)}{\theta} - \frac{(T-M_j)^2}{2} + \frac{T}{2\theta} - \frac{M_j^2}{2\theta T}\right)\right\}$$

-----------------------------(7)

$$\Pi_{j2} = ((1-k_j\Lambda)C_b - C_v - (1-k_j\Lambda)C_b I_{v0}M_j + k_j(1-\Lambda)C_b fvc\,(M2 - M1))(a+\frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}) - \frac{S_v}{nT} - C_v(I_v + I_{v0})[(n-1)(1-\varrho)+\varrho]\left(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}\right) + \left(C_c - (1-k_j\Lambda)C_b\right)(a+\frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}) - \frac{S_b}{T} - (1-k_j\Lambda)c_b I_b(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}) - (1-k_j\Lambda)C_b\left(\frac{a\theta T}{2} + \frac{b\theta T}{2}\right) + \frac{I_{be}C_c}{T}\left[\frac{aT^2}{2} + \frac{bT^3}{3}\right] + I_{be}C_c(M_j - T)(a+\frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2})$$

-----(8)

The optimum value of cycle time can be obtained by setting $\frac{d\Pi_j}{dT} = 0$ for fixed n. The necessary condition for maximising total profit is $\frac{d^2\Pi_j}{dT^2} < 0$.

5. Numerical examples

To illustrate the above developed model, an inventory system with the following data is considered a=1000, b= 50 , θ=0.1, ϱ=0.7, C_v= \$5/unit, C_b= \$35/ unit, C_c = \$ 55 / unit, S_v =\$1000/setup, S_b= \$50/order, I_v= 1%/unit/annum, I_b=1%/unit/annum, I_{v0}= 2%/unit/annum, I_{b0}= 5%/unit/annum, I_{be} = 8%/unit/annum, M_1 = 10days, M2 = 30 days, Λ = 2% and fvc =0.17/\$/annum

Using computational procedure optimum cycle time T* for above data is 21 days for n = 5. The buyer's order quantity Q* are 91,945 units/order. Vendor's total profit TVP is $21,950 and buyer's total profit TBP is $ 2,51,150. The maximum total joint profit of the integrated system п is $2,73,100.

5.1 Sensitivity analysis

Sensitivity analysis of the integrated system with respect to parameters: demand scale parameter, demand rate parameter, deterioration rate and capacity utilisation is presented in table 1, table 2 , table 3, and table 4. In each analysis the base parameter values are as assumed in Example1 and only the parameter of interest is varied holding all other parameter constant.

Table 1: Sensitive analysis for the demand scale parameter

Parameter a	T(days)	Q	Vendor	Buyer	Joint Profit
1,500	17	91,268	27,175	3,24,030	3,51,200
2,000	16	98,920	33,083	3,99,750	4,32,840
3,000	14	1,04,770	44,069	5,57,510	6,01,570

Table 2: Sensitive analysis for the demand rate parameter

Parameter b	T(days)	Q	Vendor	Buyer	Joint Profit
60	20	1,14,220	24,500	2,73,400	2,97,900
75	21	1,46,530	28,217	3,07,900	3,36,120
100	22	1,98,550	34,269	3,67,240	4,01,510

Table 3: Sensitive analysis for the deterioration rate

Parameter ө	T(days)	Q	Vendor	Buyer	Joint Profit
0.15	19	1,81,880	31,189	2,98,160	3,29,350
0.2	20	4,73,980	42,365	3,43,080	3,85,450
0.3	20	24,62,600	60,840	4,34,910	4,95,750

Table 4: Sensitive analysis for the capacity utilisation

Parameter ϱ	T(days)	Q	Vendor	Buyer	Joint Profit
0.6	**19**	**91,945**	**20,689**	**2,51,150**	**2,71,840**
0.8	**19**	**91,945**	**23,211**	**2,51,150**	**2,74,360**
0.9	**19**	**91,945**	**24,472**	**2,51,150**	**2,75,620**

From table 1 it is observed that as demand scale parameter increases vendor's profit, buyer's profit and joint total profit of the supply chain also increases. Similarly it is shown from table 2,3,4 that as demand rate parameter, deterioration rate and ratio between production rate and the market demand rate increases joint total profit of integrated inventory system is also increases. From table 4 we can conclude that if there is a change in capacity utilisation parameter then only vendor's total profit changes, buyer's total profit and order quantity remain same. Above table shows that Profit gains in percentage are positive for the entire supply chain. Therefore two part trade credit is beneficial to the supply chain as a whole.

6 Conclusion

In this paper, we first formulated an integrated vendor–buyer inventory model with the assumptions that the market demand is time dependent, units in inventory deteriorate at a constant rate and the vendor offers two payment options: trade credit and early-payments with discount price to the buyer. A solution procedure is constructed to compute cycle time and order quantity which maximizes the integrated profit. Numerical examples are given to validate the proposed model. It is concluded that a two – part trade credit offer can increase profits of the buyer, vendor and the entire supply chain. It is observed that as the vendor and buyer take joint decision, the channel profit will increase significantly.

As for future research, our model can be extended to more general supply chain networks, for example, multi-echelon supply chains. Also, it is interested to consider different deteriorating rates of items into the proposed model and consider the order quantity as a function of credit period.

7 References

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